

PHOTOPTICS 2020

8th International Conference on Photonics, Optics and Laser Technology

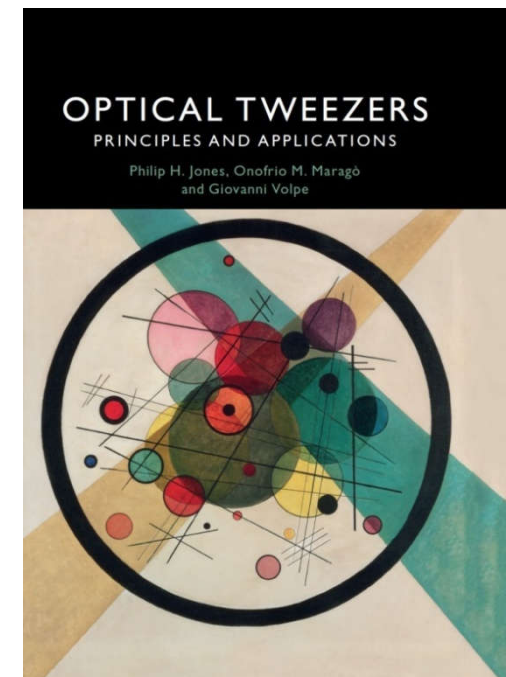
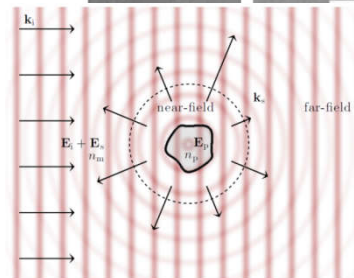
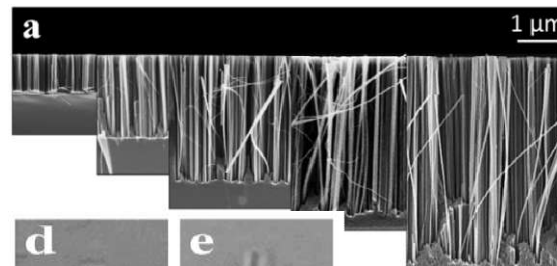
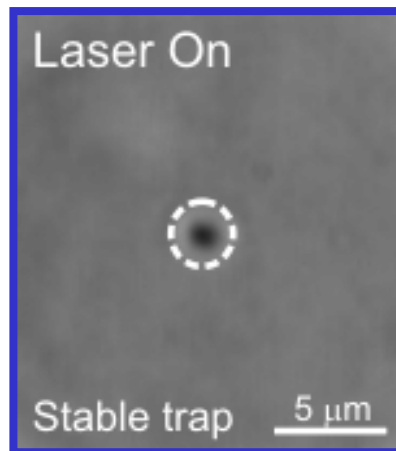
VALLETTA - MALTA | 27 - 29 FEBRUARY, 2020

Optical Tweezers on Nanostructures

Onofrio M. MARAGÒ

CNR-IPCF, Istituto per i Processi Chimico-Fisici, Messina, Italy

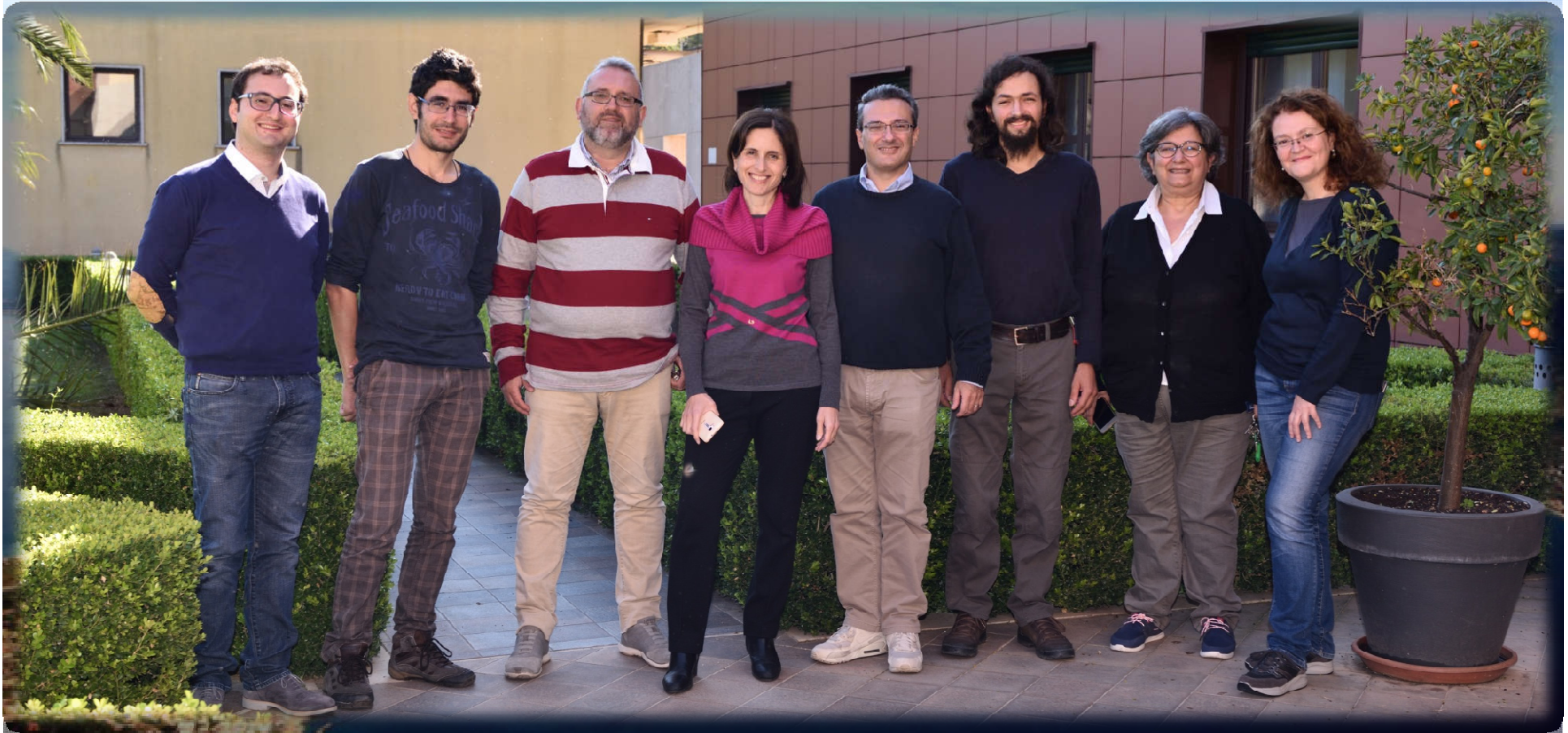
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CNR-IPCF Messina

Optical Tweezers People - Messina 2018



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O. M. Maragò (CNR), R. Gillibert (Post-Doc), R. Saija (UniMe), M.G. Donato (CNR)

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Acknowledgements

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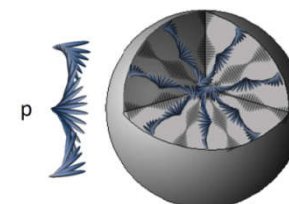
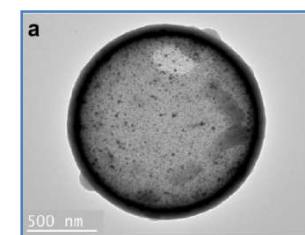
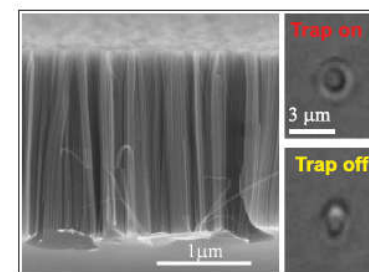
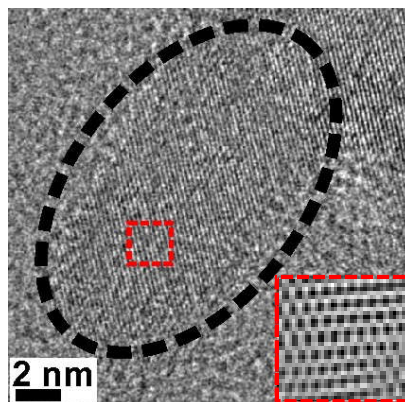
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G. Cipparrone (Univ. Calabria)

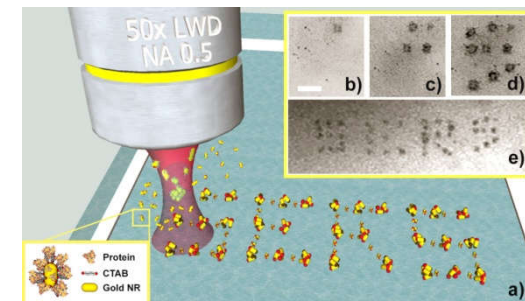
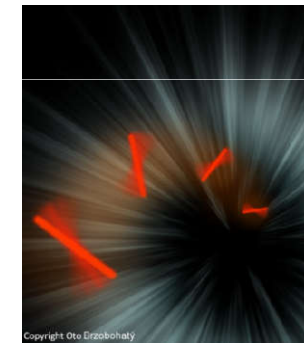
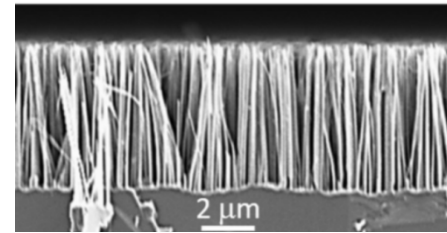
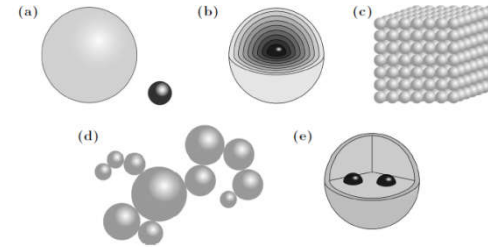
A. Neves (Univ. ABC, Brasil)

G. Pesce (Univ. Napoli)

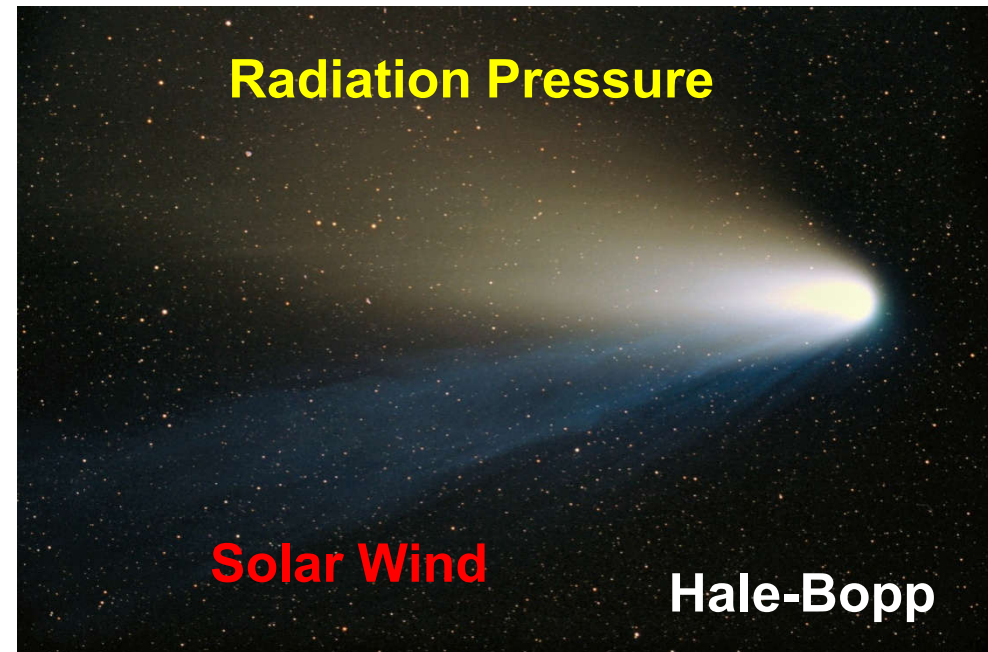
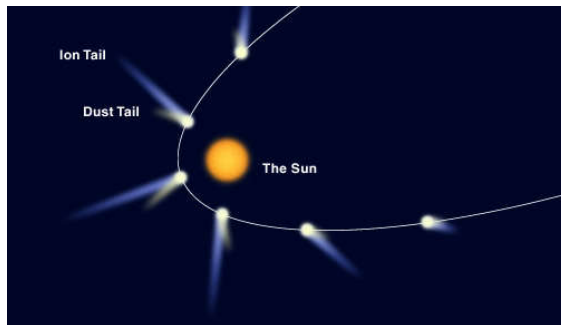
A. Veltri (USFQ, Quito)



- **General introduction**
- **Theory background**
 - *T-matrix methods*
- **Experimental practice**
- **Linear nanostructures**
 - *Scaling, Binding & rotational dynamics*
 - *Air and vacuum*
- **Layered materials**
- **SERS Tweezers**




- **J Kepler (1610)**
comet tails are the result of light pressure



- **J C Maxwell (1864)**
light pressure is explained in electromagnetic theory
- **P Lebedev (1901) and Nichols & Hull (1901)**
measures light pressure for the first time
- **A Ashkin, T Haensch & A Schawlow, V Lethokov (1970s)**
first proposals to manipulate atoms and microparticles, laser cooling
- **A Ashkin & S Chu (1986)**
at Bell Laboratories moves and traps latex spheres suspended in water using a focused laser beam. **Optical Tweezers** are born!

Illustrations: Niklas Elmehed

THE NOBEL PRIZE IN PHYSICS 2018

Three stylized line drawings of the Nobel laureates. Arthur Ashkin is on the left, with a beard and short hair. Gérard Mourou is in the center, wearing glasses and having long hair. Donna Strickland is on the right, wearing glasses and having shoulder-length hair. The drawings are done in a sketchy style with gold and black ink on a light background.

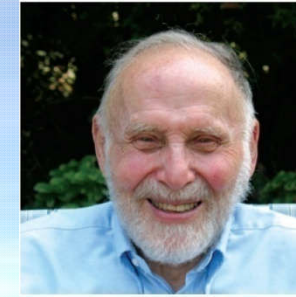
Arthur
Ashkin

Gérard
Mourou

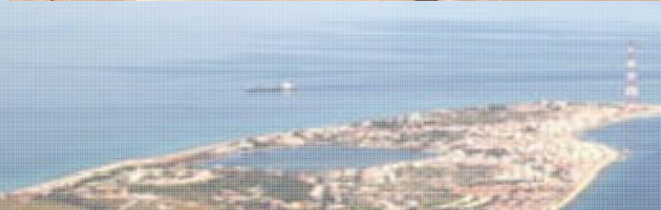
Donna
Strickland

“for groundbreaking inventions
in the field of laser physics”

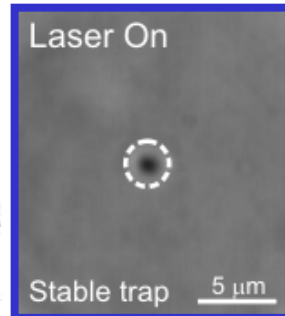
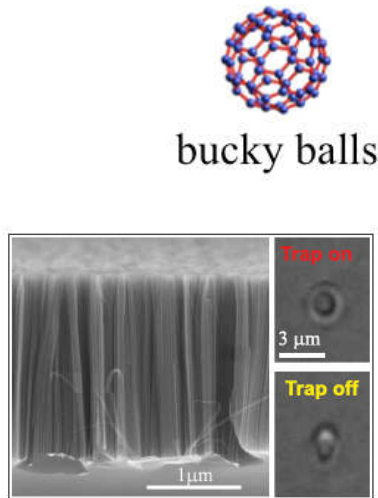
THE ROYAL SWEDISH ACADEMY OF SCIENCES



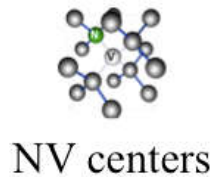
Celebrations for Arthur Ashkin's Nobel prize 2018



Atom Trapping



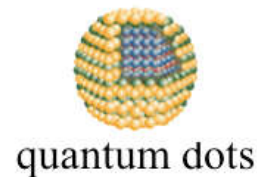
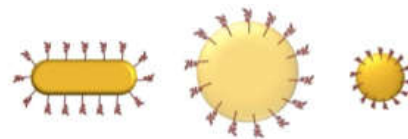
nanowires and nanotubes



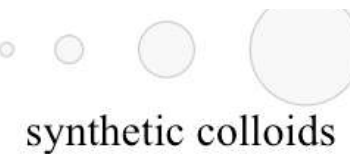
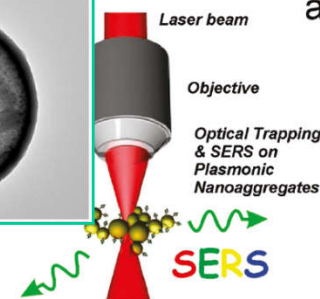
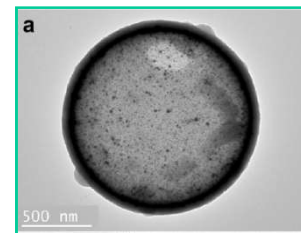
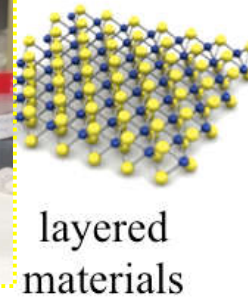
Nanotweezers



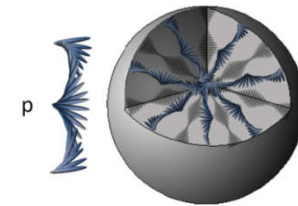
graphene



Optical Tweezers



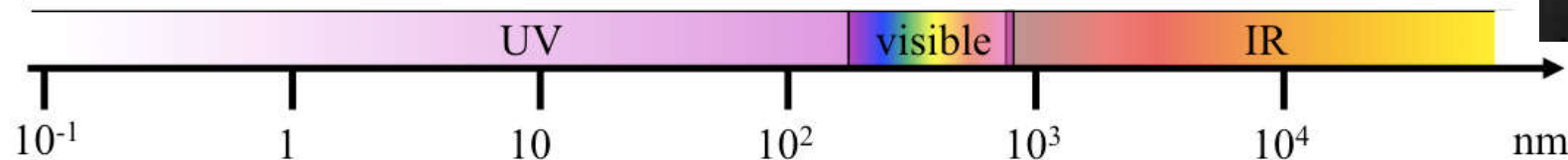
Supramolecular Chiral particles



Left-handed



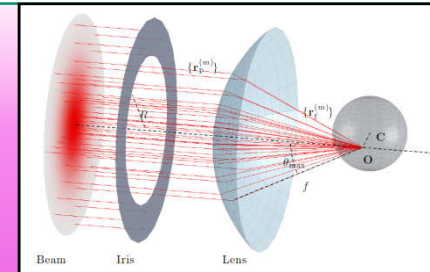
Right-handed



Optical trapping of particles is a consequence of the radiation force that stems from the **conservation of electromagnetic momentum in light scattering**.

Ray Optics, $d/\lambda \gg 1$

- Trapping forces from **reflection and refraction of rays**
- Forces proportional to **gradient of intensity**
(Ashkin, Biophys. J. 61, 569, 1992)



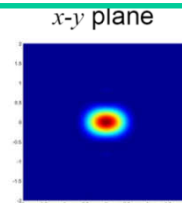
Dipole Approximation, $d/\lambda \ll 1$

- Three parts: **gradient force**, **scattering force**
(Ashkin, et al. Optics Lett. 11, 288, 1986)

$$U_{\text{dip}} = -\underline{p} \cdot \underline{E}$$

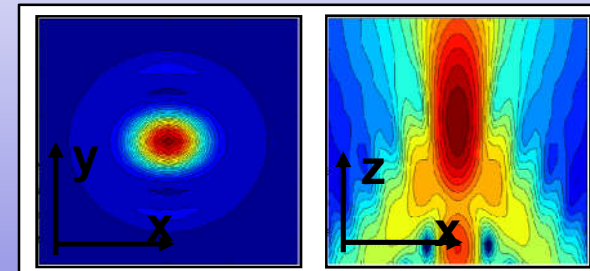
$$\underline{F}_{\text{grad}} = -\underline{\nabla} U_{\text{dip}}$$

$$\propto \underline{\nabla} I(\underline{r}) = -\kappa_i x_i$$



Complex Region, $d/\lambda \approx 1$

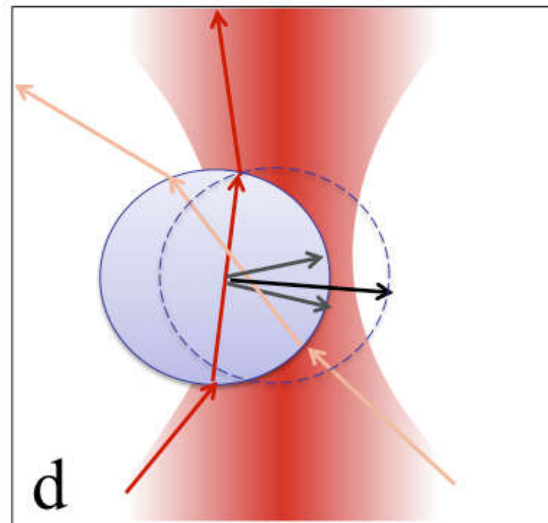
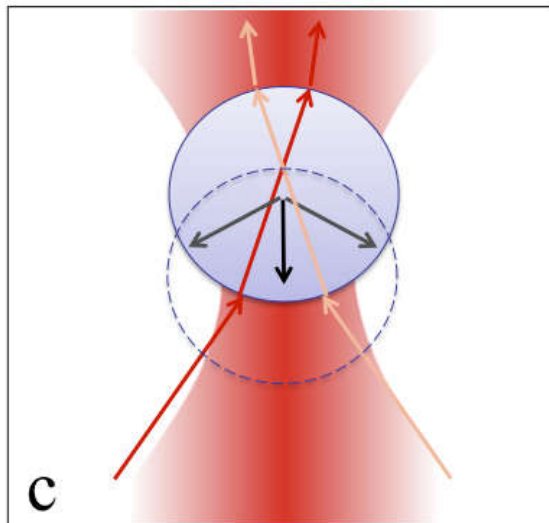
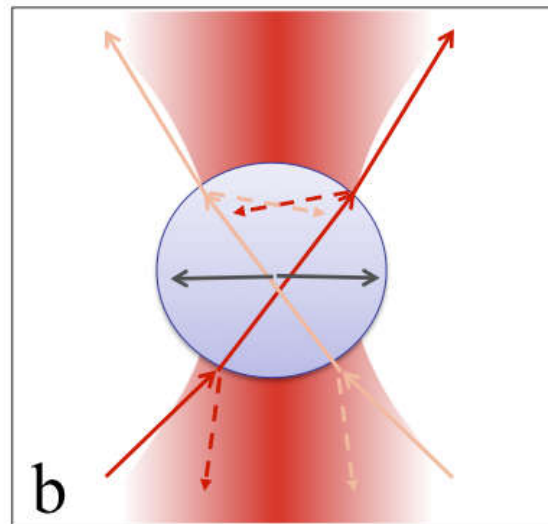
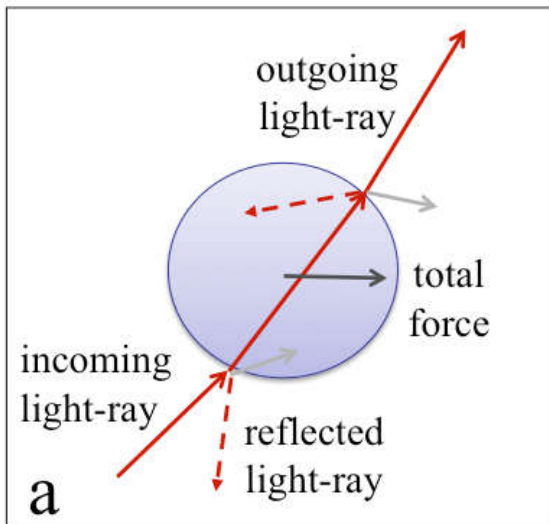
- Full electromagnetic Theory (**Maxwell ST**)
- Make use of **T-Matrix methods** for force & torque
- **Angular spectrum representation**



Extension to complex non-spherical particles

$$\mathbf{F}_{\text{Rad}} = r'^2 \int_{\Omega'} \hat{\mathbf{r}}' \cdot \langle \mathbf{T}_M \rangle d\Omega'$$

Borghese, Denti, Saija, Springer (2007)
 Borghese et al., Optics Express (2007)
 Borghese et al., Phys Rev Lett (2008)
 Saija et al., Opt. Express (2009) ...



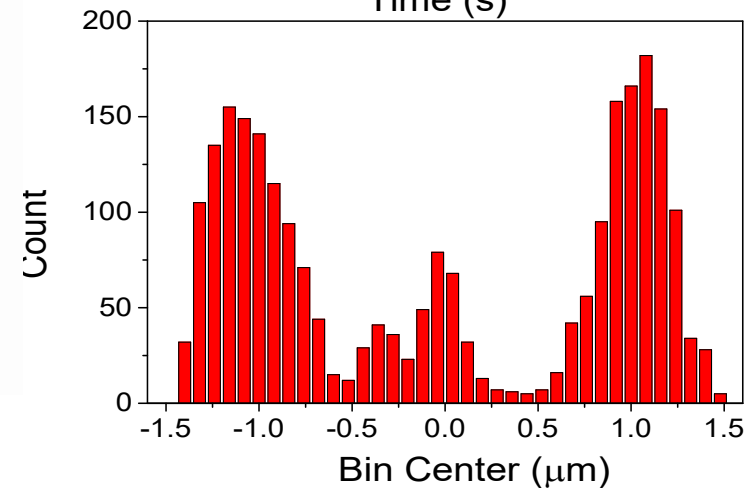
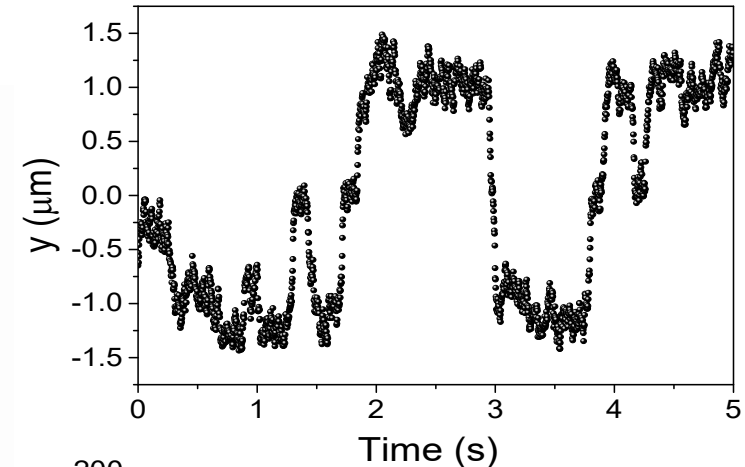
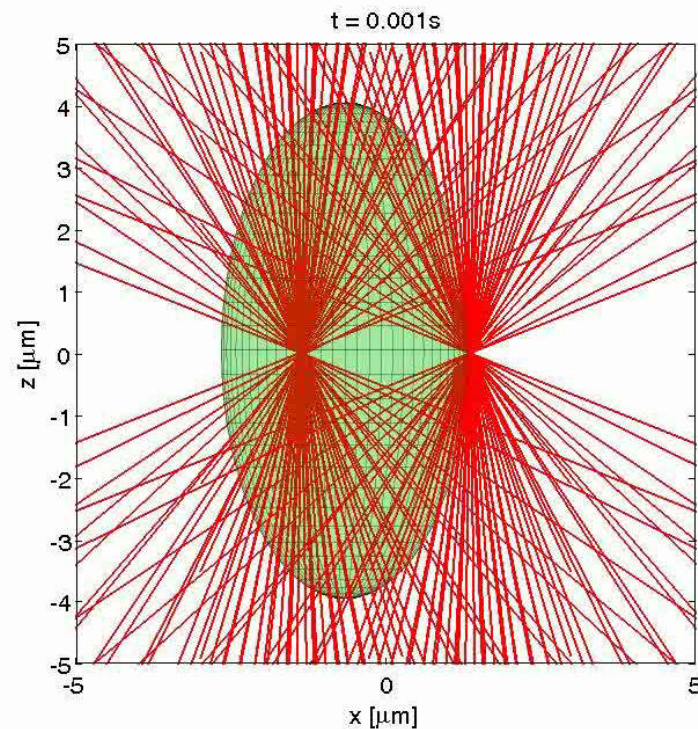
Microsphere acts as a lens for the refracted and reflected rays

Linear momentum exchange from light to particle, push towards beam center ($n_p > n_m$)

Trapping Force is proportional to the gradient of light intensity

Scattering force is proportional to light intensity and directed as the Poynting vector

$$\mathbf{F} = \sum_m \mathbf{F}^{(m)} = \sum_m \left[\frac{P_i^{(m)}}{c_i} \hat{\mathbf{u}}_i^{(m)} - \frac{P_r^{(m)}}{c_i} \hat{\mathbf{u}}_{r,0}^{(m)} - \sum_{n=1}^{+\infty} \frac{P_{t,n}^{(m)}}{c_i} \hat{\mathbf{u}}_{t,n}^{(m)} \right]$$



Diffusion Matrix Modeling for complex particles using Hydro++

G. Volpe & G. Volpe American Journal of Physics **81**, 224 (2013).

A. Callegari, M. Mijalkov, et al. JOSA B **32**, B11-B19 (2015)

A. Stilgoe et al., Phys. Rev. Lett. **107**, 248101 (2011).

By Magazzù, Callegari

Interaction of electric field of laser with induced dipole in dielectric $U = -\underline{p} \cdot \underline{E} = -\alpha |E|^2$

Average force on particle $\langle \mathbf{F} \rangle = \frac{1}{2} \text{Re} \left(\sum_i \alpha E_i \nabla E_i^* \right)$ For a Gaussian beam breaks in two contributions

$$F_{\text{scatt}} = \hbar k \frac{\Gamma}{2} \frac{I/I_{\text{sat}}}{1 + I/I_{\text{sat}} + 4\delta^2/\Gamma^2}$$

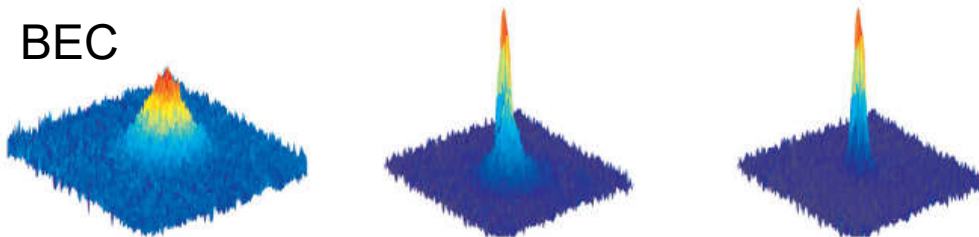
$$F_{\text{dipole}} = -\frac{\hbar\delta}{2} \frac{\nabla(I/I_{\text{sat}})}{1 + I/I_{\text{sat}} + 4\delta^2/\Gamma^2}$$

$$F_{\text{scat}}(r) = \frac{n\sigma_{\text{ext}}}{c} I(r)$$

$$F_{\text{grad}}(r) = \frac{n \text{Re}\{\alpha(\omega)\}}{2c\epsilon} \nabla I(r)$$

The (harmonic) trapping potential is defined by the incident light intensity

BEC

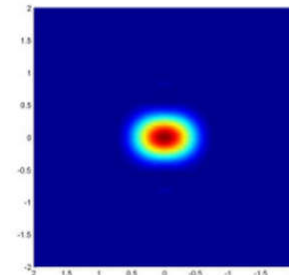


$$U_{\text{dip}} = -\underline{p} \cdot \underline{E}$$

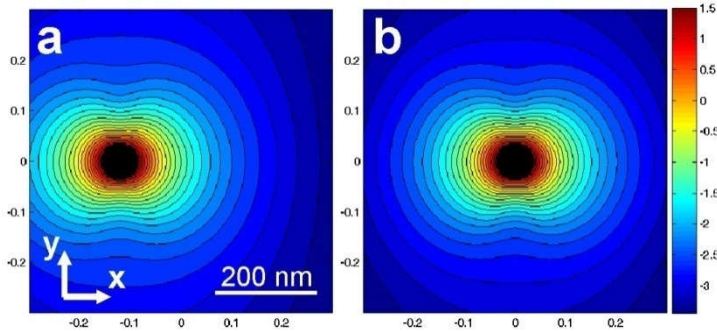
$$F_{\text{grad}} = -\underline{\nabla} U_{\text{dip}}$$

$$\propto \underline{\nabla} I(\underline{r}) = -\kappa_i x_i$$

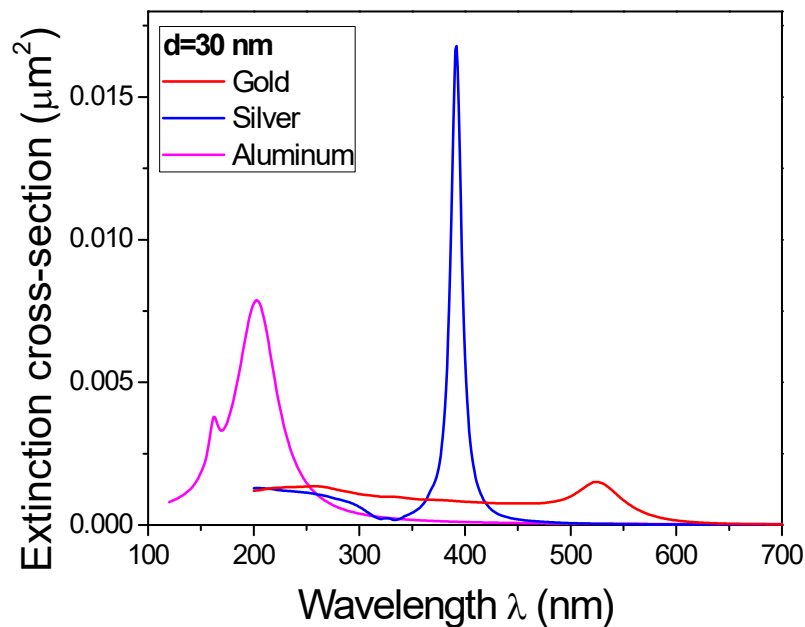
x-y plane



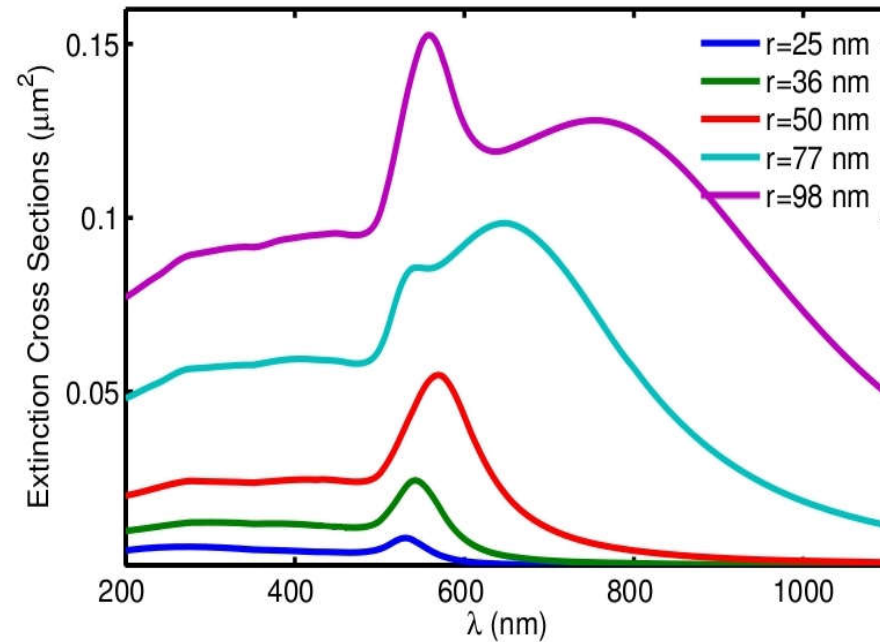
Optical response of colloidal metal nanoparticles



For **metal nanoparticles**, the presence of **plasmon resonances** leads to their optical trapping with a wavelength in the **red side** of the spectrum.



Amendola, V., et al. (2017). Surface plasmon resonance in gold nanoparticles: a review. *J. Phys.: Cond. Matt.*, **29**, 203002.



Saija R et al. *Optics Express* (2009)
 Jones et al., *ACS Nano* (2009) – Au Nanorods
 Messina et al., *Optics Express* (2015) – Ag Platelets

Optical trapping of plasmonic particles (dipole picture)

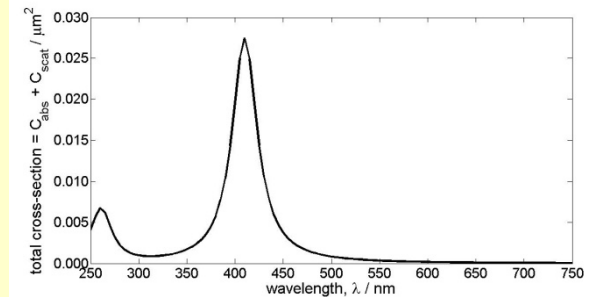


Lorentz-Drude model for dielectric constant using tabulated fit parameters that agrees well with experimental data and exploit plasmon resonance to enhance optical forces

$$\epsilon(\omega) = \epsilon_\infty + \sum_{k=1}^K \frac{f_k \omega_p^2}{\omega_k^2 - \omega^2 + i\omega\Gamma_k}$$

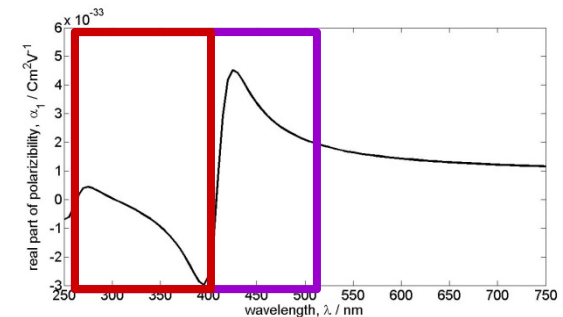
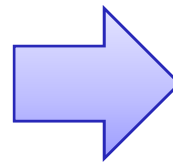
$$\alpha(\omega) = 4\pi\epsilon_0 a^3 \frac{\epsilon_1(\omega) - \epsilon_2}{\epsilon_1(\omega) + 2\epsilon_2}$$

$$\sigma_{abs} = \frac{k}{\epsilon_0} \text{Im}\{\alpha(\omega)\}; \quad \sigma_{scat} = \frac{k^4}{6\pi\epsilon_0^2} |\alpha(\omega)|^2$$



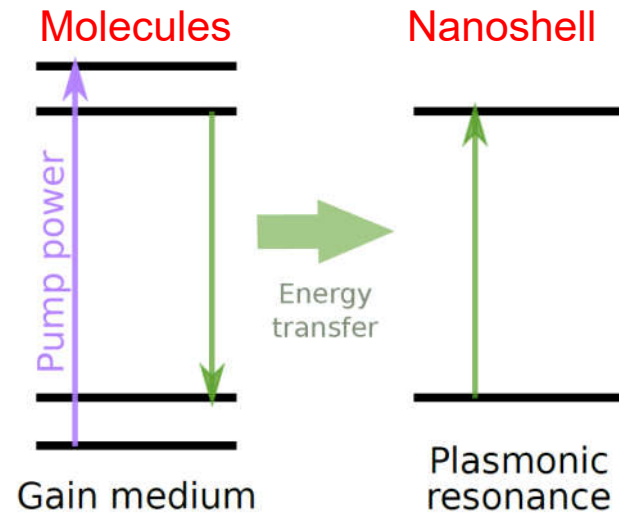
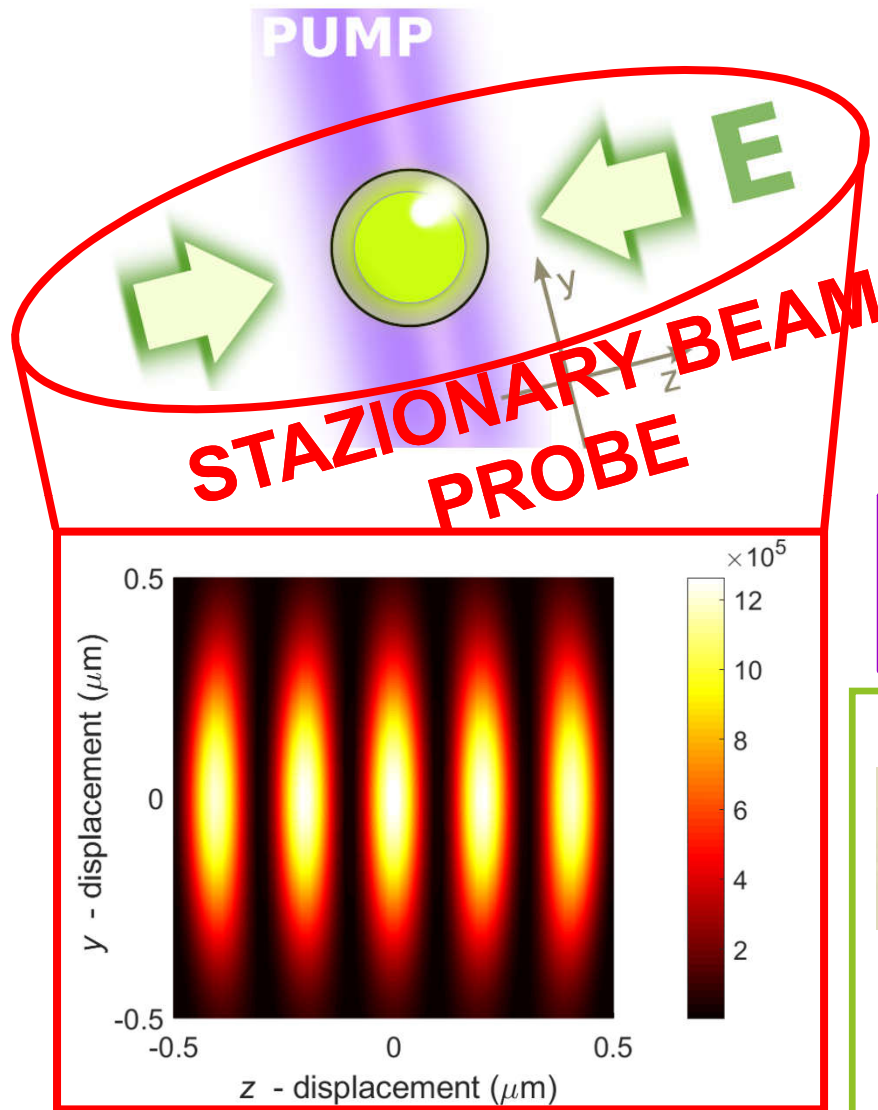
$$F_{scat}(r) = \frac{n\sigma_{ext}}{c} I(r)$$

$$F_{grad} = \frac{n}{2} \frac{\text{Re}\{\alpha(\omega)\}}{c\epsilon_0} \nabla I(r)$$



Trap Stiffness proportional to $\text{Re}\{\alpha(\omega)\}$

Trapped resonant gain metal/dielectric nanoshell



Silver nanoshell with externally pumped optical gain material on its core excited by an external electric field.

Steady State Gain dielectric permittivity

$$\epsilon_1 = \epsilon_b - \frac{G\Delta}{2(\omega - \omega_{21}) + i\Delta} \quad \Delta = \frac{2}{\tau_2}$$

ϵ_b dielectric host permittivity

$$G = \Im[\epsilon_1(\omega_{21})] = -\frac{n\mu^2\tau_2}{3\hbar\epsilon_0} \tilde{N}$$

Brownian Dynamics Simulation

G. Volpe and G. Volpe,
Am. J. Phys. (2013)

$$\frac{d}{dt}r(t) = -\frac{1}{\gamma} \frac{d}{dr}U(r) + \xi(t)$$

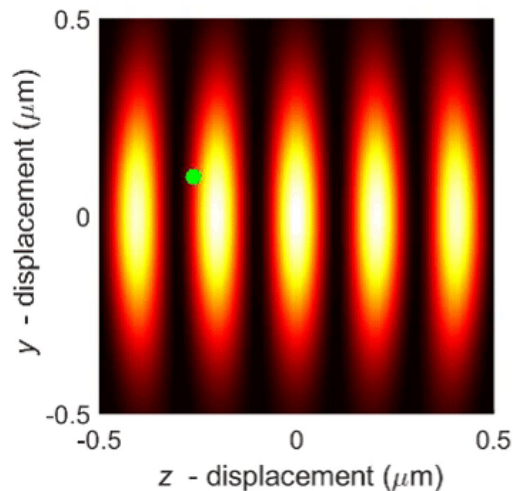
Overdamped
Langevin equation

Integrating

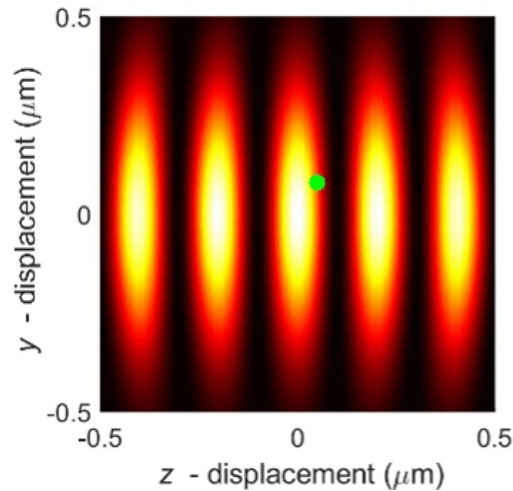
$$\begin{cases} x_i = x_{i-1} - \frac{\kappa_x}{\gamma} x_{i-1} \Delta t + \sqrt{2D\Delta t} w_{x,i} \\ y_i = y_{i-1} - \frac{\kappa_y}{\gamma} y_{i-1} \Delta t + \sqrt{2D\Delta t} w_{y,i} \\ z_i = z_{i-1} - \frac{\kappa_z}{\gamma} z_{i-1} \Delta t + \sqrt{2D\Delta t} w_{z,i} \end{cases}$$

$P = 20$ mW
 $t = 1$ ms
 $\Delta t = 2$ ns

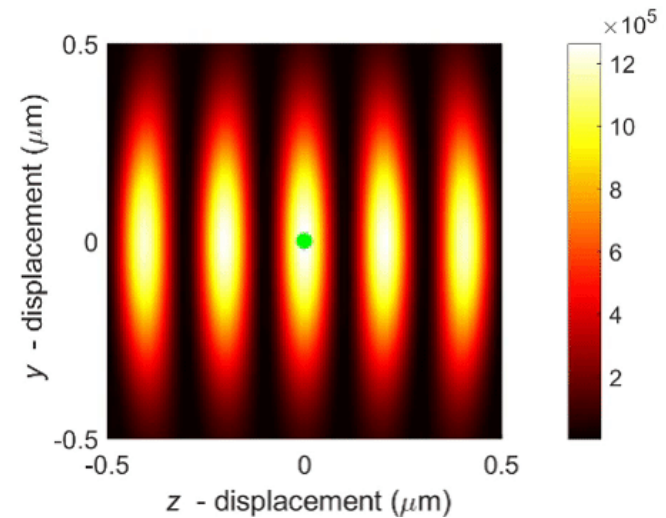
$G = -0.022$



$G = -0.132$

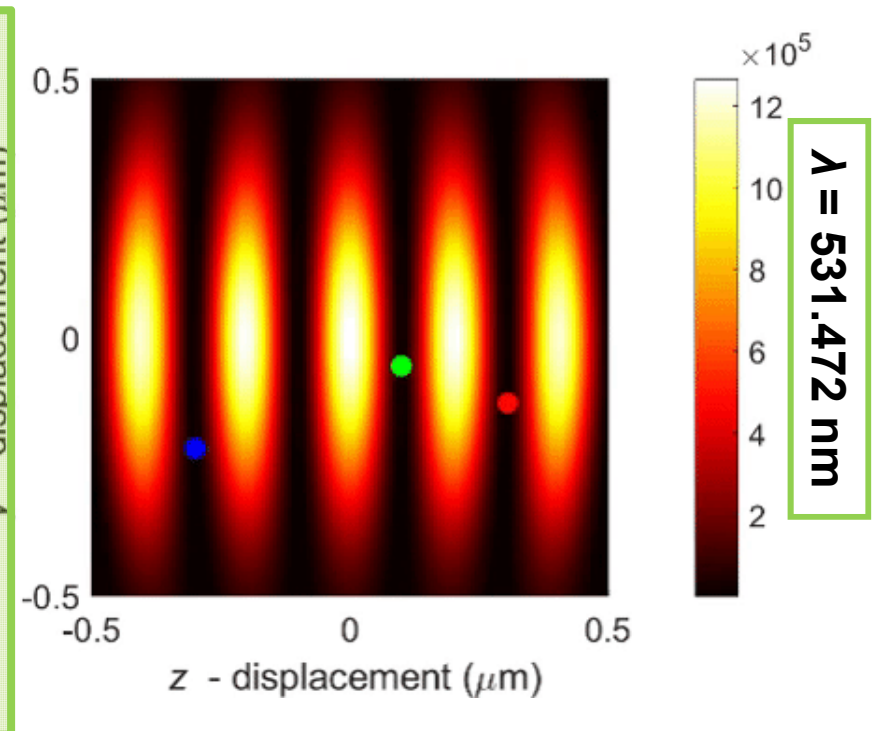
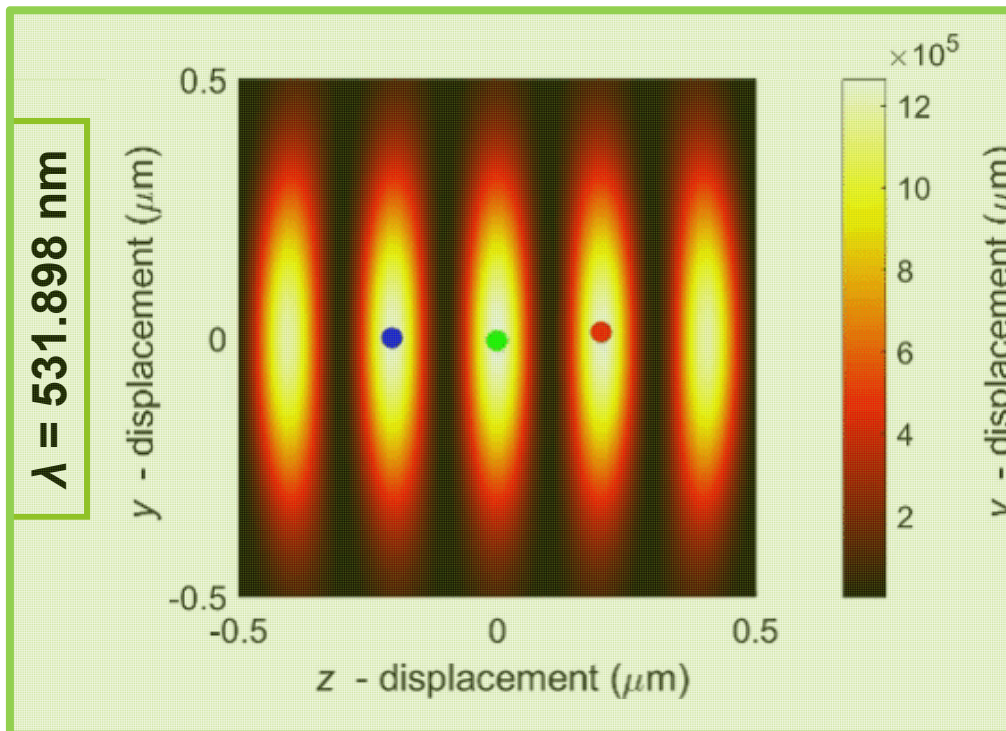
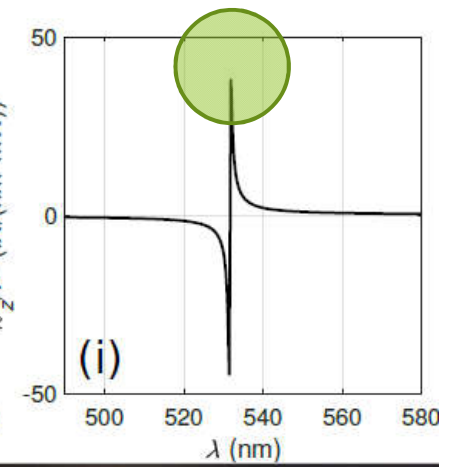
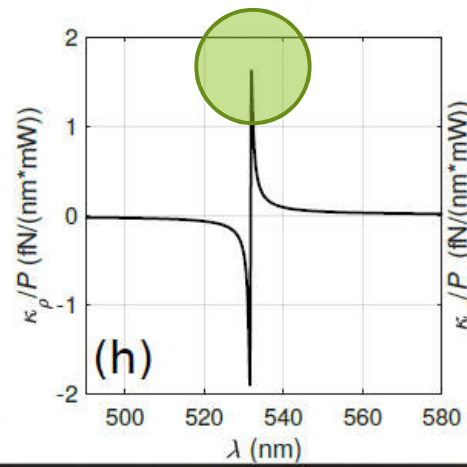
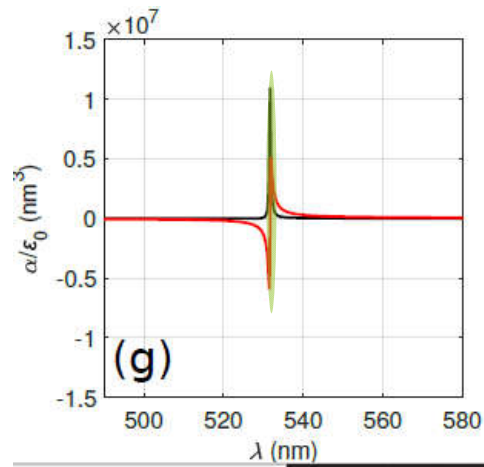


$G = -0.22$



Optomechanical Position Locking and Channelling

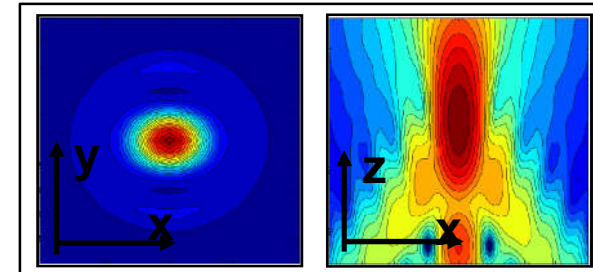
$G = -0,22$
 $P = 20 \text{ mW}$
 $t = 1 \text{ ms}$
 $\Delta t = 2 \text{ ns}$



Optical trapping of particles is a consequence of the **conservation of electromagnetic momentum** and the **spatial redistribution of photons** in light scattering.

Mesoscopic Region, $d/\lambda \approx 1$

- Full electromagnetic Theory
- Vector character of laser field
- Make use of Transition-Matrix approach
- **Extension to non-spherical particles!**



Conservation of linear momentum

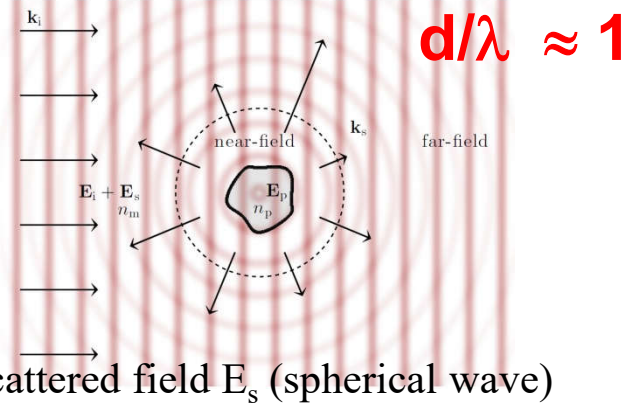
$$\left\langle \frac{d\mathbf{P}_{Mech}}{dt} \right\rangle = \mathbf{F}_{Rad} = \oint_S \hat{\mathbf{n}} \cdot \langle \mathbf{T}_M \rangle dS$$

Conservation of angular momentum

$$\left\langle \frac{d\mathbf{L}_{Mech}}{dt} \right\rangle = \Gamma_{Rad} = - \oint_S \hat{\mathbf{n}} \cdot \langle \mathbf{T}_M \rangle \times \hat{\mathbf{n}} dS$$

$$\mathbf{E}_t = \mathbf{E}_i + \mathbf{E}_s$$

Incident field \mathbf{E}_i
(plane wave)



Scattered field \mathbf{E}_s (spherical wave)

Averaged Maxwell stress tensor

$$\langle \mathbf{T}_M \rangle = \frac{1}{2} \epsilon_m \Re \left\{ \mathbf{E}_t \otimes \mathbf{E}_t^* + \frac{c^2}{n_m^2} \mathbf{B}_t \otimes \mathbf{B}_t^* - \frac{1}{2} \left(|\mathbf{E}_t|^2 + \frac{c^2}{n_m^2} |\mathbf{B}_t|^2 \right) \mathbf{I} \right\}$$

$$\begin{cases} a_l = -\frac{A_{s,lm}^{(2)}}{W_{i,lm}^{(2)}} \\ b_l = -\frac{A_{s,lm}^{(1)}}{W_{i,lm}^{(1)}} \end{cases}$$

Boundary conditions on a sphere
Only l -dependence
T-matrix is **diagonal**

Radiation pressure (plane wave)

$$\mathbf{F}_{\text{rad}}^{\parallel} = \frac{n_m}{c} I_i [\sigma_{\text{ext}} - g_i \sigma_{\text{scat}}] \hat{\mathbf{k}}_i$$

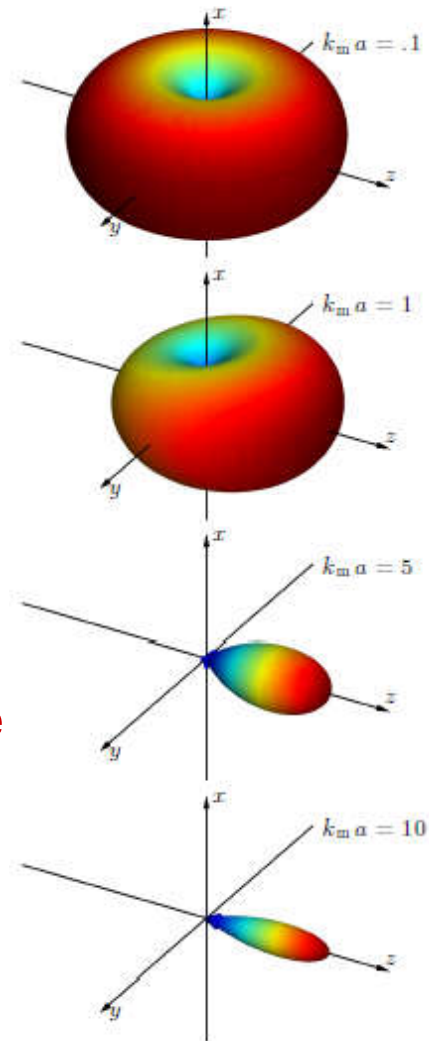
Debye (1909), Mishchenko (2001)

$$\sigma_{\text{ext}} = \frac{2\pi}{k_m^2} \sum_{l=1}^{\infty} (2l+1) \text{Re} \{a_l + b_l\}$$

$$\sigma_{\text{scat}} = \frac{2\pi}{k_m^2} \sum_{l=1}^{\infty} (2l+1) (|a_l|^2 + |b_l|^2)$$

$$g_i = \frac{4\pi}{\sigma_{\text{scat}} k_m^2} \text{Re} \left\{ \sum_{l=1}^{+\infty} \left[\frac{l(l+2)}{l+1} (a_l a_{l+1}^* + b_l b_{l+1}^*) + \frac{2l+1}{l(l+1)} a_l b_l^* \right] \right\}$$

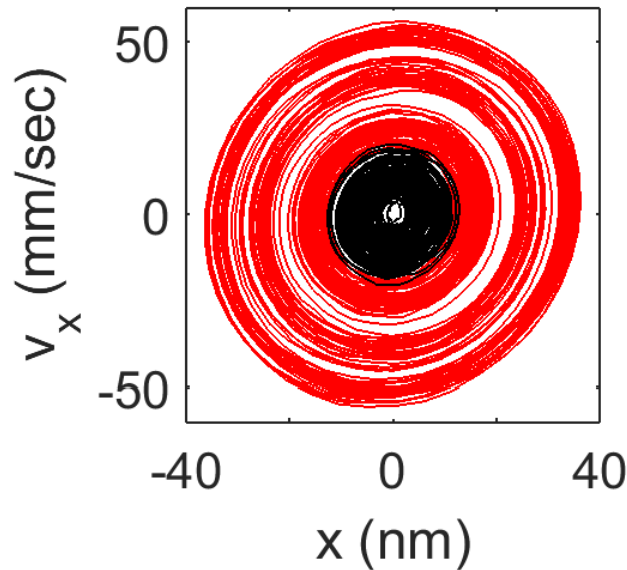
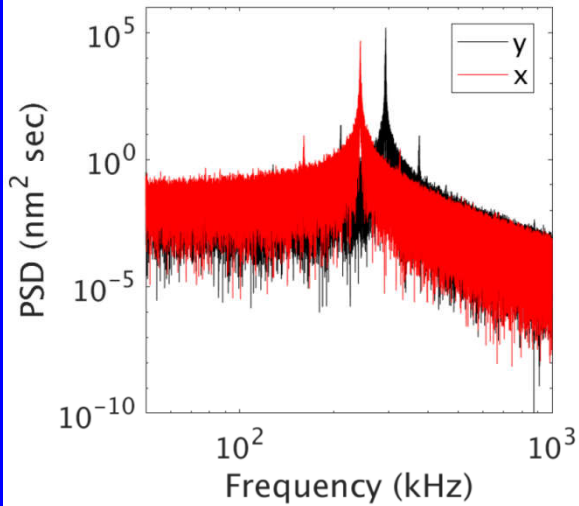
Observables can be
obtained from the
Mie coefficients



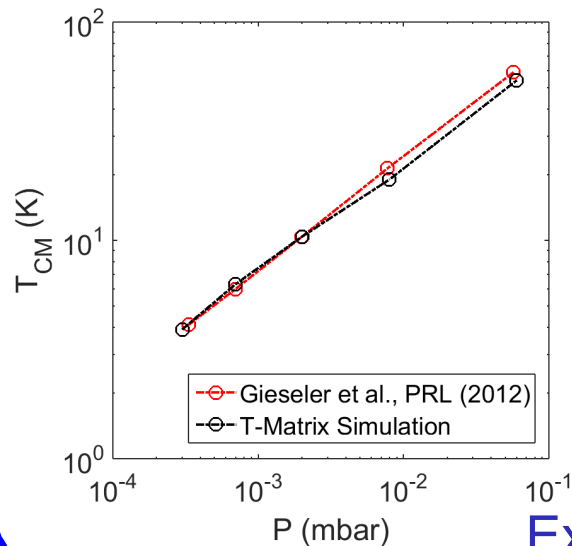
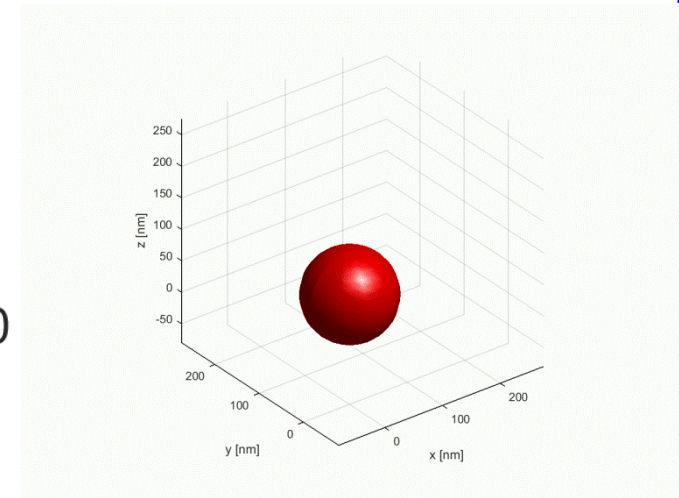
T-matrix & Brownian dynamics

Modelling feedback cooling

Cooling in phase-space (feedback on x)



Feedback off-on



$$F_{FB} = \beta \frac{r v_r}{\langle r \rangle_{\beta=0} \langle v_r \rangle_{\beta=0}} F_x$$

See also Levi et al. Cont. Phys. (2014)

Extension to non-spherical particles

Mechanical effects on non-spherical particles

Radiation Pressure } Conservation of
Transverse Force } linear momentum

$$\mathbf{F}_{\text{rad}}^{\parallel} = \frac{n_m}{c} I_i [\sigma_{\text{ext}} - g_i \sigma_{\text{scat}}] \hat{\mathbf{k}}_i = \frac{n_m}{c} I_i \sigma_{\text{rad}} \hat{\mathbf{k}}_i$$

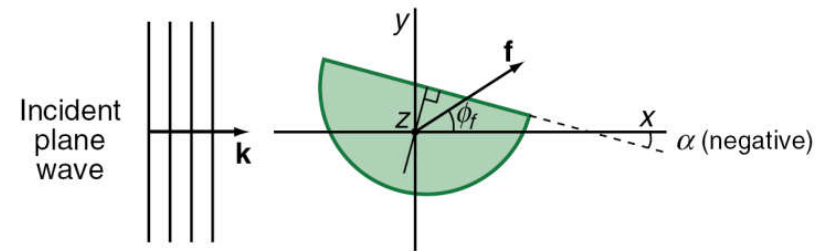
$$\mathbf{F}_{\text{rad}}^{\perp} = -\frac{n_m}{c} I_i g_1 \sigma_{\text{scat}} \hat{\mathbf{u}}_1 - \frac{n_m}{c} I_i g_2 \sigma_{\text{scat}} \hat{\mathbf{u}}_2$$

Radiation Torque → Conservation of angular momentum

$$g_i = \frac{1}{\sigma_{\text{scat}}} \oint_{\Omega} \frac{d\sigma_{\text{scat}}}{d\Omega} \hat{\mathbf{r}} \cdot \hat{\mathbf{k}}_i d\Omega$$

Saija et al, MNRAS, 2003

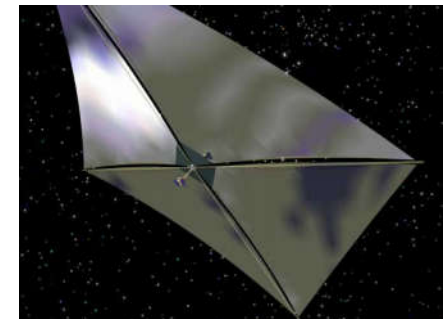
Radiation cross-sections yield the mechanical effects of light on non-spherical particles



Wind sailing



Solar sailing

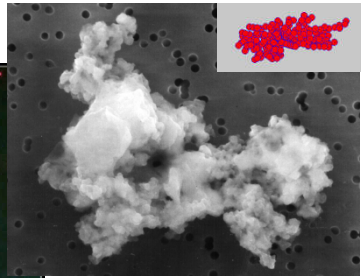


Swartzlander, Optical lift, Nature Photonics 2010

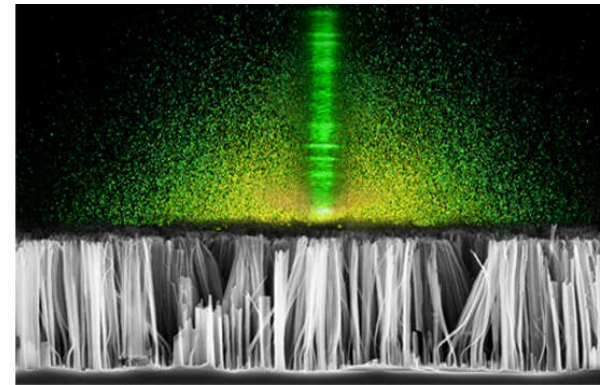
Simpson et al, Opt Lett **37**, 2012



Fazio et al, Light: Science & Applications (2016)
 Fazio et al. Nature Photonics (2017)



Astrophysics:
 Interstellar dust

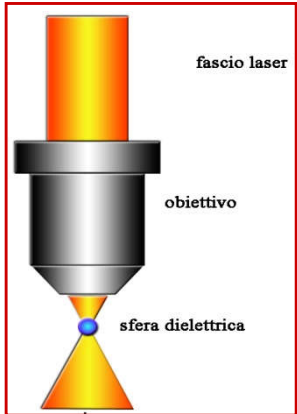


Scattering and light
 localization in disordered
 systems

- Iati et al., MNRAS (2001)
- Saija et al., MNRAS (2003)
- Iati et al., ApJ (2004)
- Iati et al., MNRAS (2008)

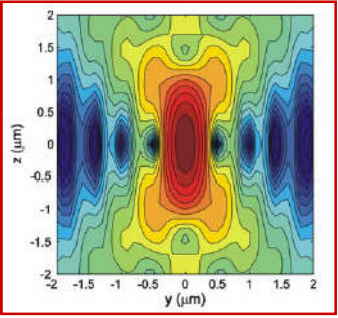
T-Matrix Methods

Optical tweezers

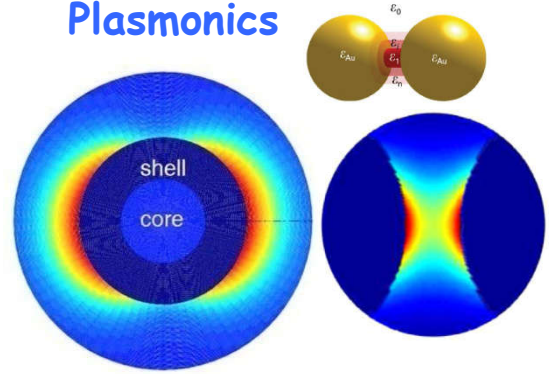


Borghese et al., Optics
 Express (2007)

Borghese et al., PRL (2008)

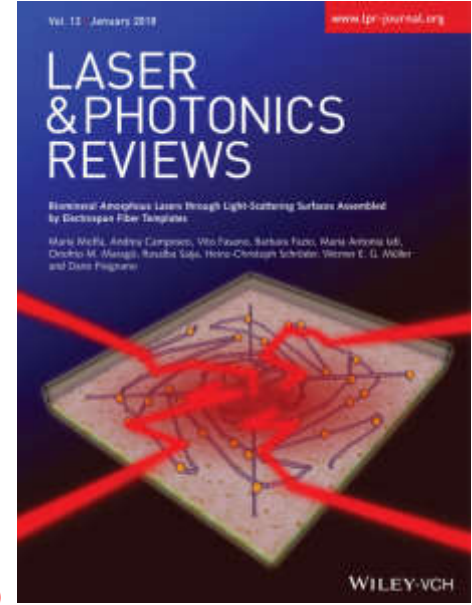


Plasmonics



Review: Amendola et al. J Phys: Cond Matt (2017)

Cacciola et al., JQSRT (2017)



Moffa et al.,
 Laser & Photonics Reviews (2018)

Radiation force & torque from a general scattering process

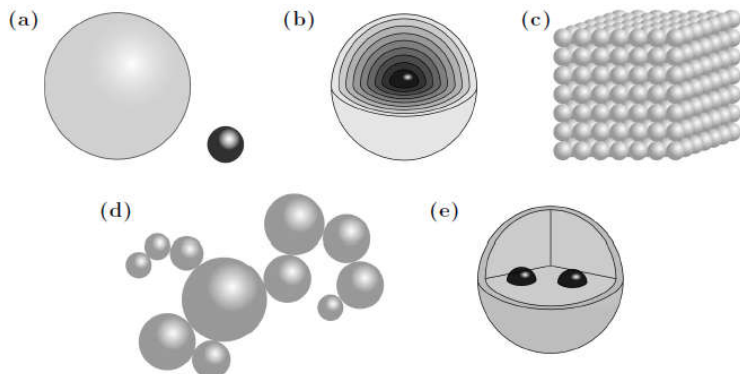
Borghese equations

Generalize Mishchenko,
JQSRT (2001)

$$F_{\text{rad}}(\hat{\mathbf{u}}) = -\frac{\epsilon_m E_i^2}{2k_m^2} \text{Re} \left\{ \sum_{plm} \sum_{p'l'm'} i^{l-l'} I_{lm'l'm'}^{(pp')}(\hat{\mathbf{u}}) \left[\overset{\text{scattering}}{A_{s,lm}^{(p)*} A_{s,l'm'}^{(p')}} + \overset{\text{extinction}}{W_{i,lm}^{(p)*} A_{s,l'm'}^{(p')}} \right] \right\}$$

$$T_{\text{rad},z} = \underbrace{-\frac{\epsilon_m E_i^2}{2k_m^3} \sum_{plm} m \text{Re} \left\{ W_{i,lm}^{(p)} A_{s,lm}^{(p)*} \right\}}_{\text{extinction}} - \underbrace{\frac{\epsilon_m E_i^2}{2k_m^3} \sum_{plm} m |A_{s,lm}^{(p)}|^2}_{\text{scattering}}$$

Generalize
Marston&Crichton
PRA(1984)



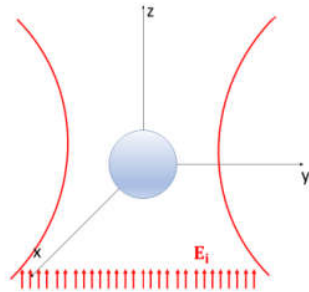
❖ **Extension to chiral particles:** See Patti, et al., Sci. Rep. (2019).

❖ **Extension to hybrid structures:** See Ridolfo et al. ACS Nano (2011)

❖ **Extension to surfaces:** See Denti et al. JOSAA (1999).

See also Saija et al. (2005), Borghese et al. (2006), Borghese et al. (2007)

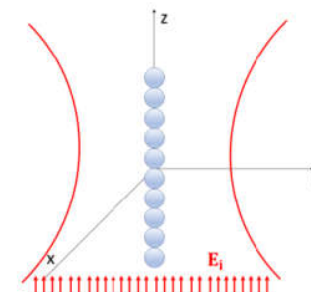
Size scaling for polystyrene spheres and nanowires



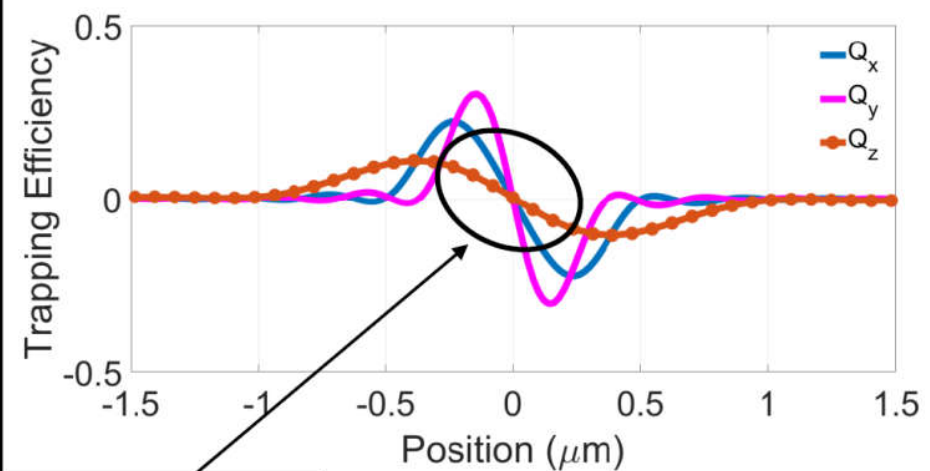
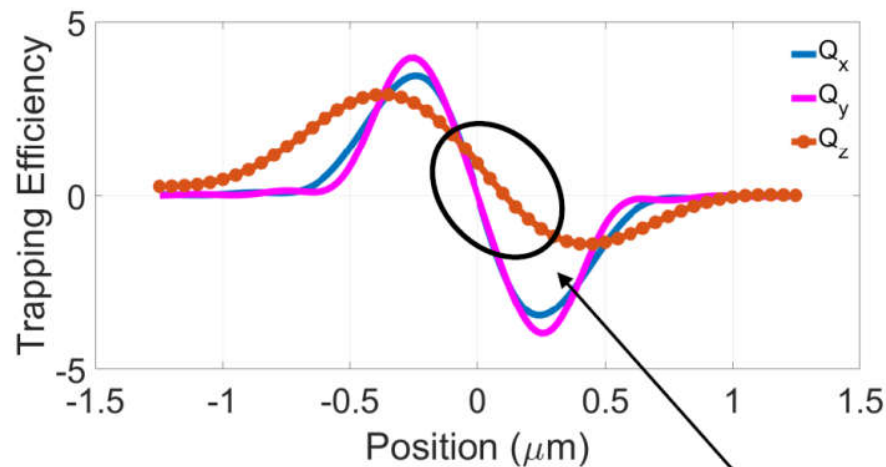
Sphere of radius
 $a = 300 \text{ nm}$

$$Q_i = F_i \frac{c}{n_m P_i}$$

$\lambda = 830 \text{ nm}$
 $\text{NA} = 1.3$
 $w_0 \approx 320 \text{ nm}$



Nanowire of half-length
 $L/2 = 300 \text{ nm}$

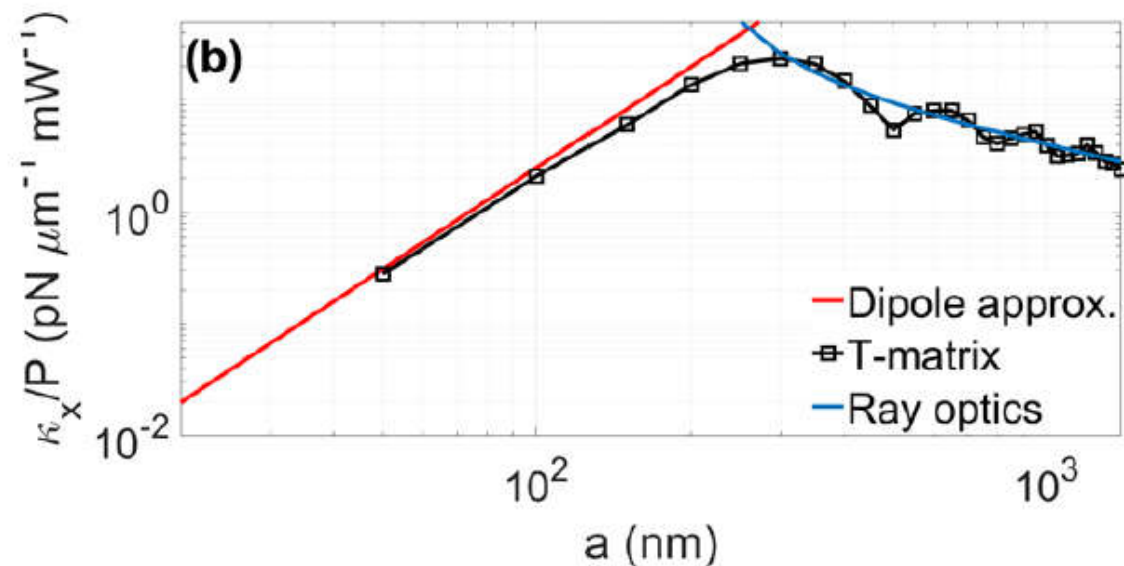
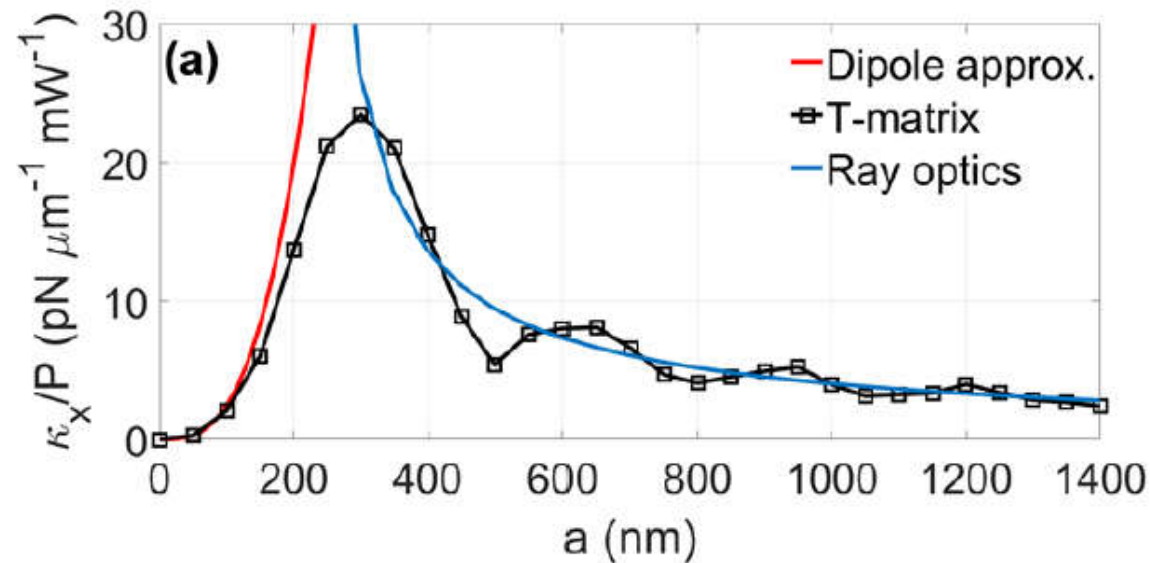


$$F_i = -\kappa_i x_i$$

Harmonic force trapping

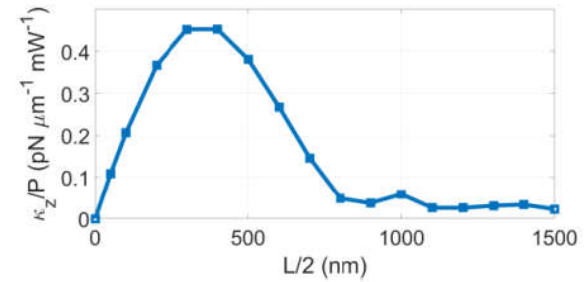
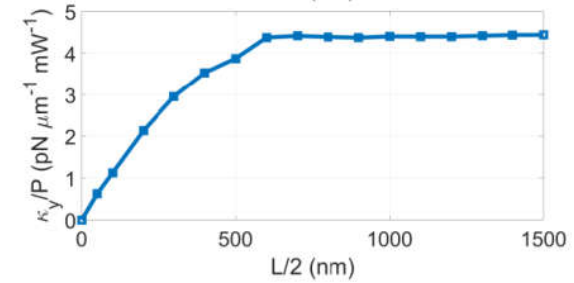
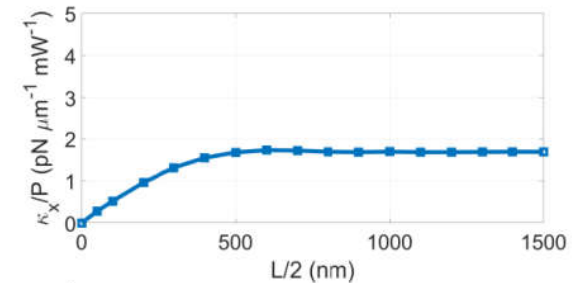
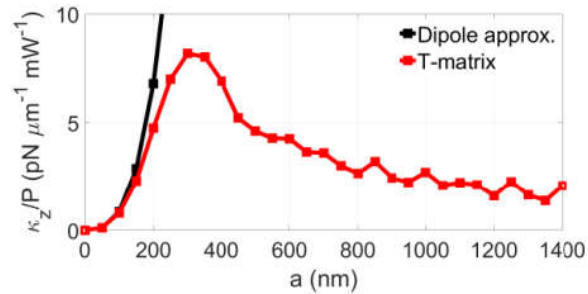
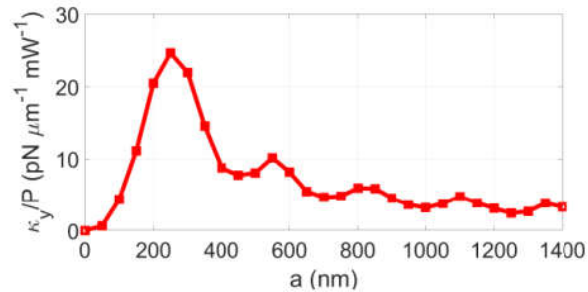
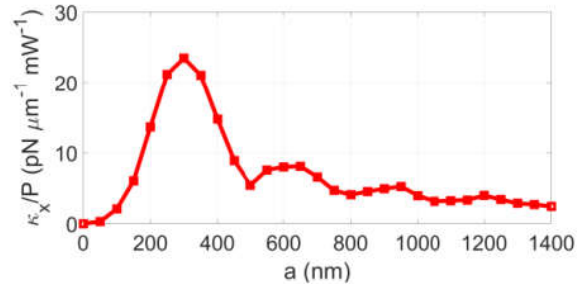
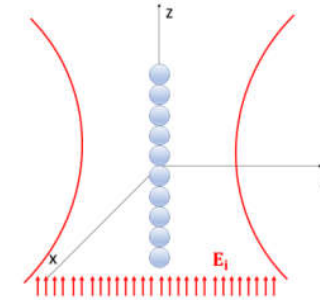
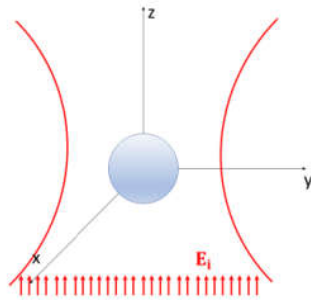


Size scaling for a sphere



Size scaling for polystyrene spheres and nanowires

$\lambda = 830 \text{ nm}$
 $NA = 1.3$
 $w_0 \approx 320 \text{ nm}$

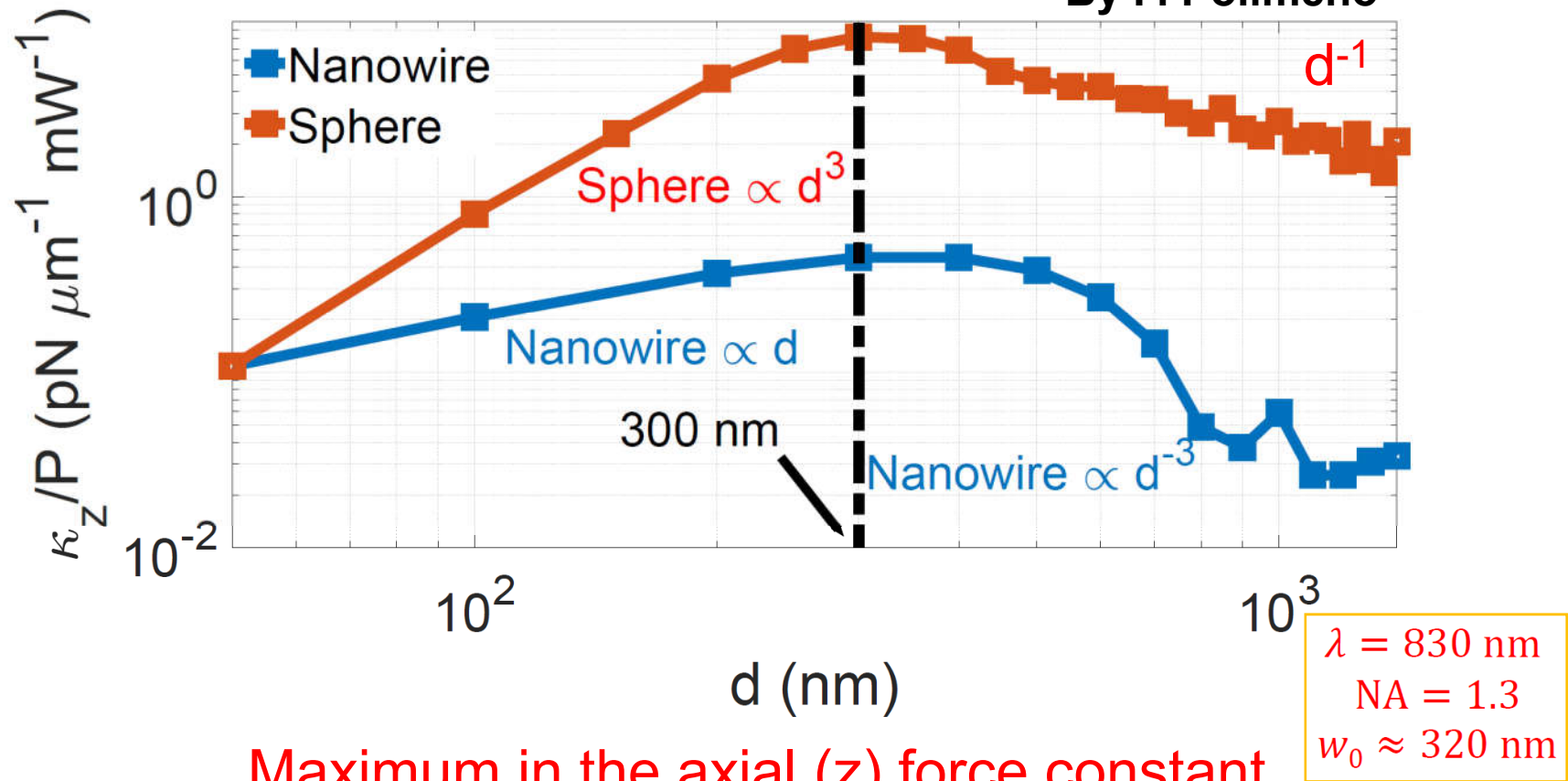


By P. Polimeno

Scaling of Force Constants

T-matrix calculations – Role of shape

By P. Polimeno



Maximum in the axial (z) force constant

Optimum overlap between wire and high intensity region (Rayleigh Range)

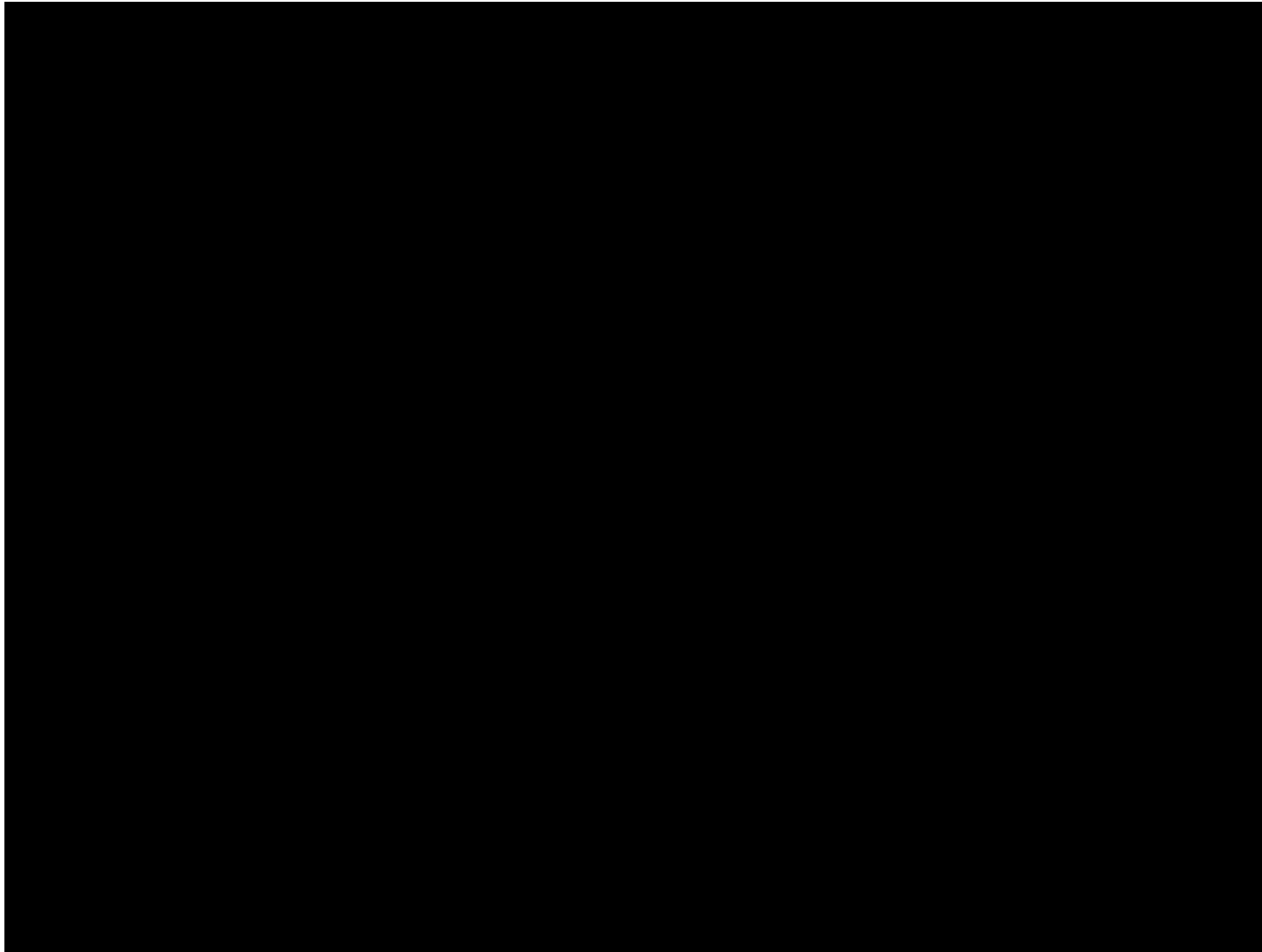
See also Simpson & Hanna, Nanotechnology (2012)



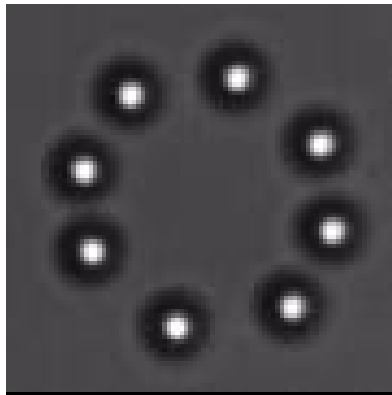
Experimental practice



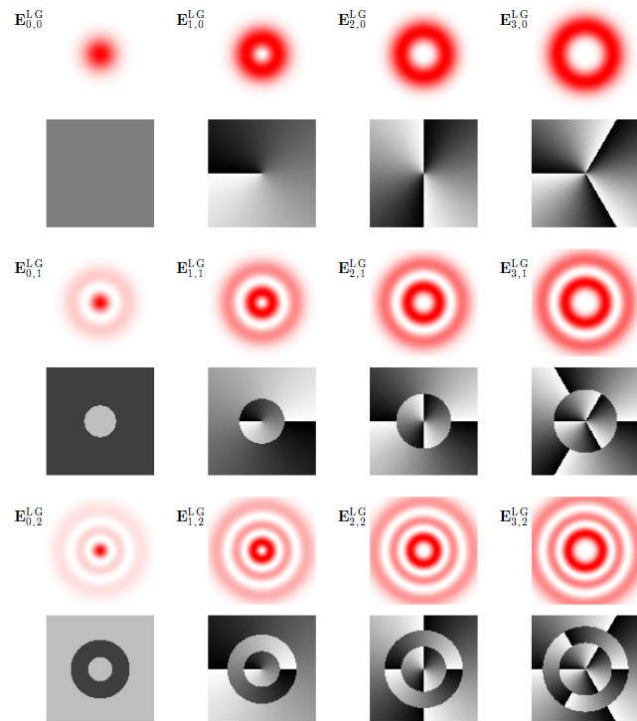
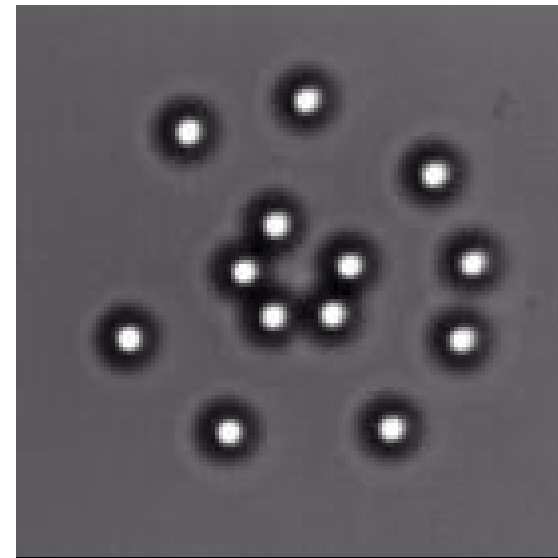
Building an Optical Tweezers



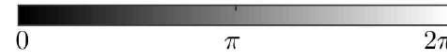
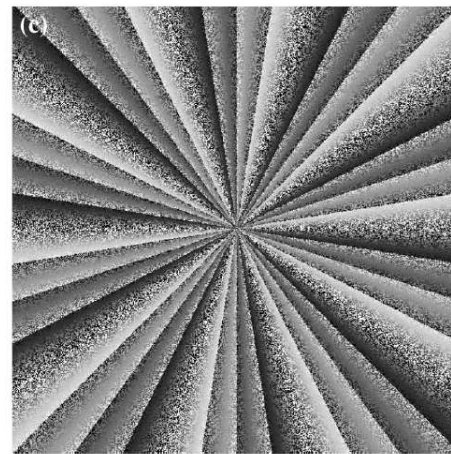
G. Pesce, et al, Step-by-step guide to the realization of advanced optical tweezers, *JOSA B* (2015)
Special Issue Opex & JOSA B on Optical Cooling & Trapping



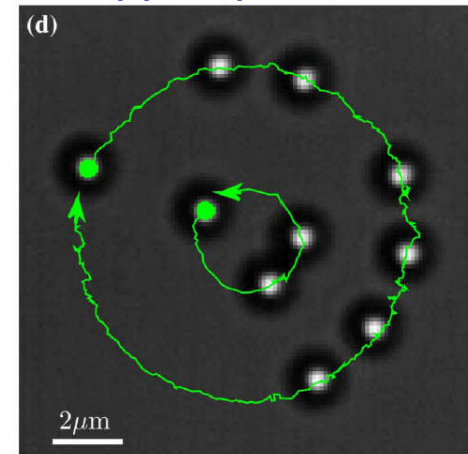
Laguerre-Gauss
Beams can transfer
Orbital Angular
Momentum



Phase Mask



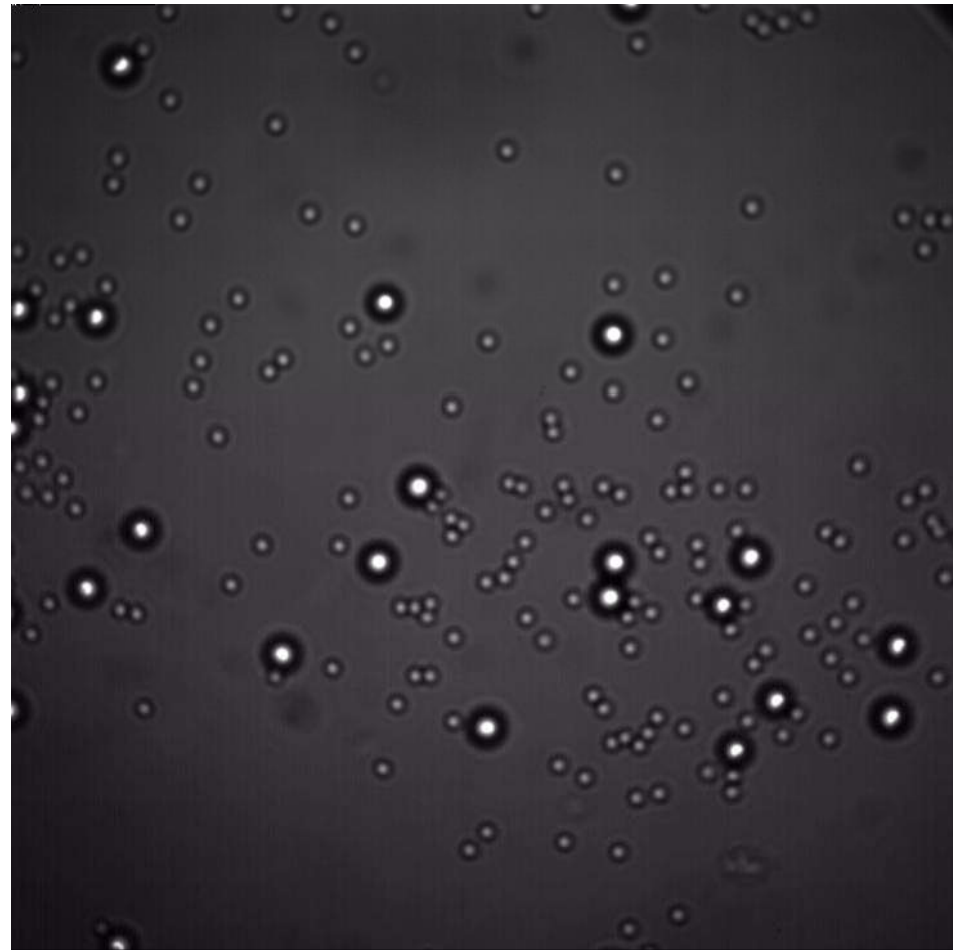
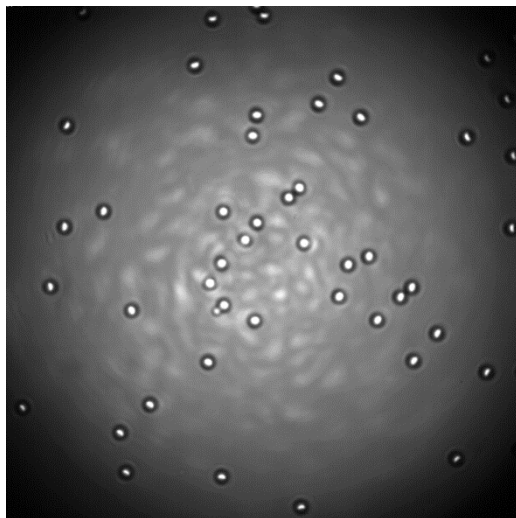
Trapped particles



Advanced Optical Tweezers (Speckle fields)

Optical sorting with speckle fields

High intensity traps large particles, while small particles move in the microfluidic flow

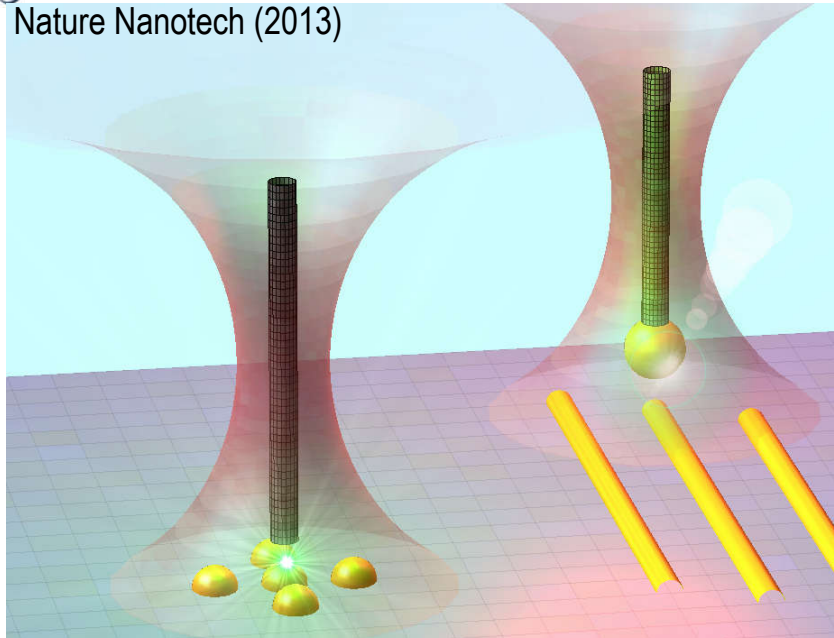




OT of Linear Nanostructures



Nature Nanotech (2013)



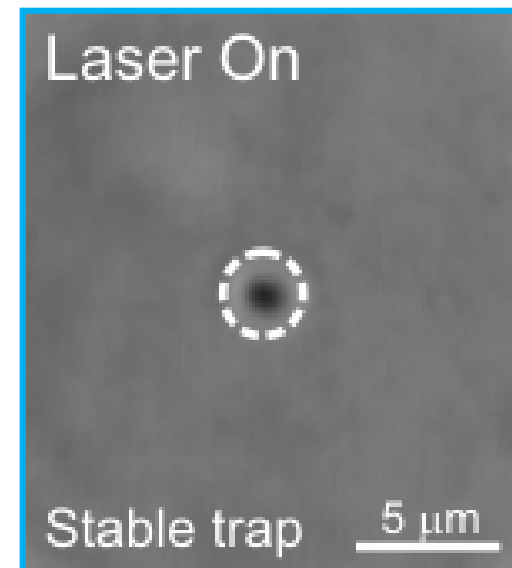
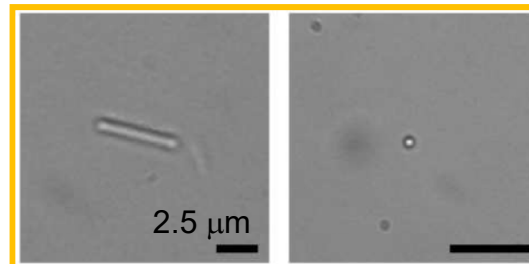
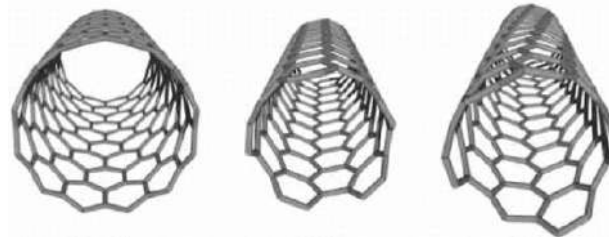
Nanowires:

- Agarwal et al., Optics Express (2005)
- Pauzauskie et al., Nat. Mat. (2006)
- Nakayama et al., Nature (2007)
- Borghese et al., Phys. Rev. Lett. (2008)*
- Carberry et al., Nanotech. (2010)
- Simpson&Hanna, JOSA A (2010)
- Simpson&Hanna, PR E (2010)
- Reece et al., Nano Lett. (2011)
- Dutta et al., Nano Lett. (2011)
- Irrera et al., Nano Lett. (2011)*

...

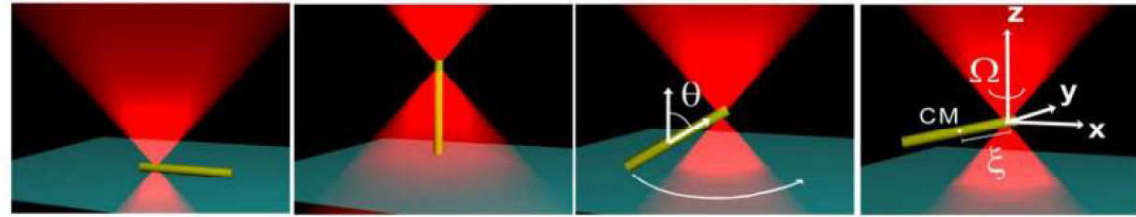
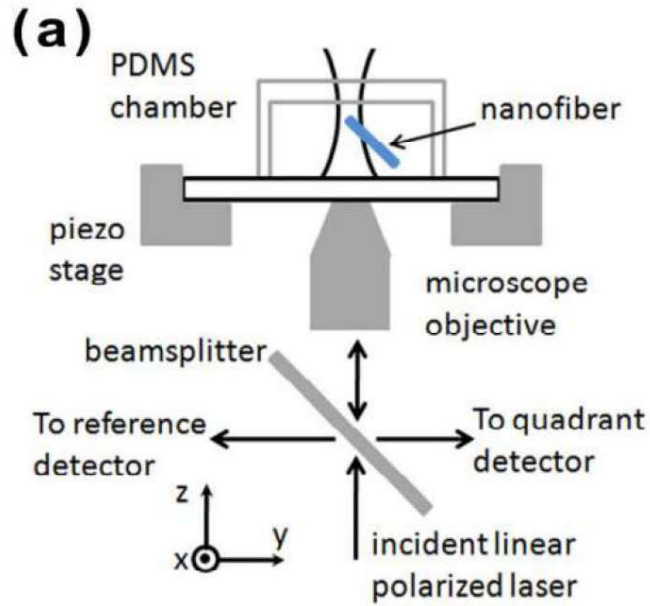
Nanotubes:

- Tan et al., Nano Lett. (2004)
- Plewa et al., Optics Express (2004)
- Zhang et al., APL (2006)
- O.M. Maragò et al., Physica E (2008)*
- O.M. Maragò et al., Nano Lett. (2008)*
- P.H. Jones, et al. ACS Nano (2009)*
- Pauzauskie et al, APL (2009)
- Donato et al., Opt. Lett. (2012)*



Nanofibers:

- Neves et al., Optics Express (2010)*

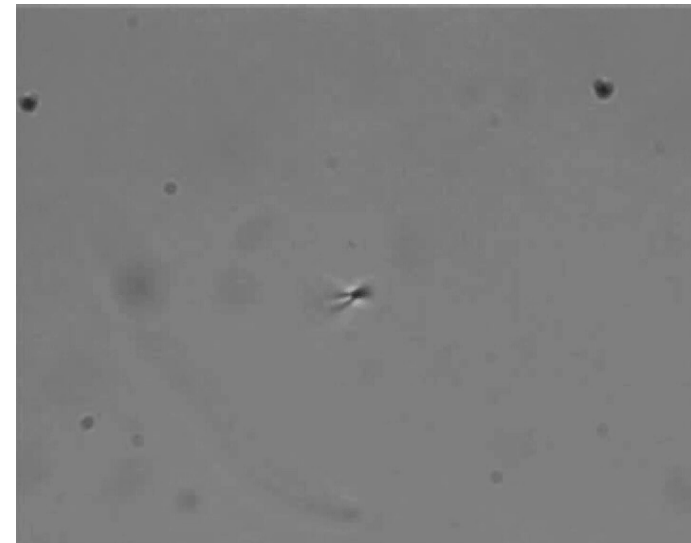
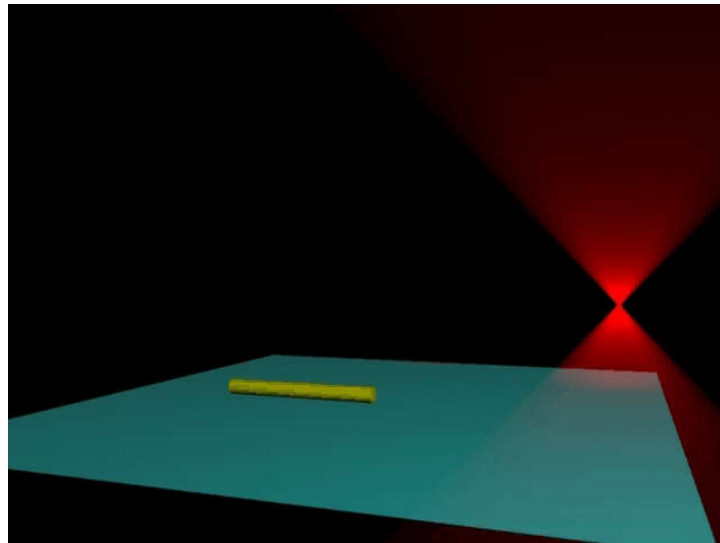
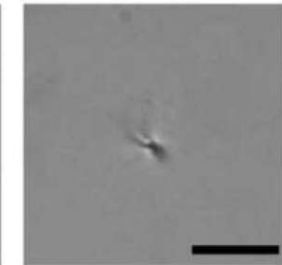
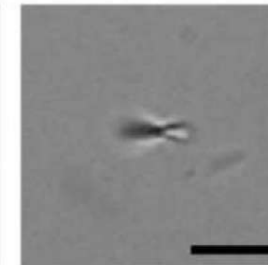
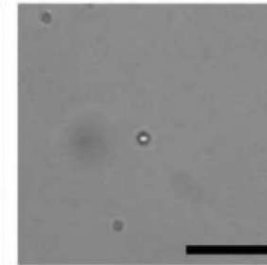
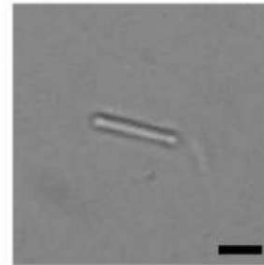


(b)

(c)

(d)

(e)

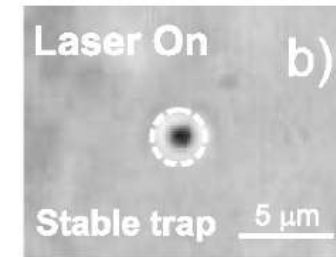
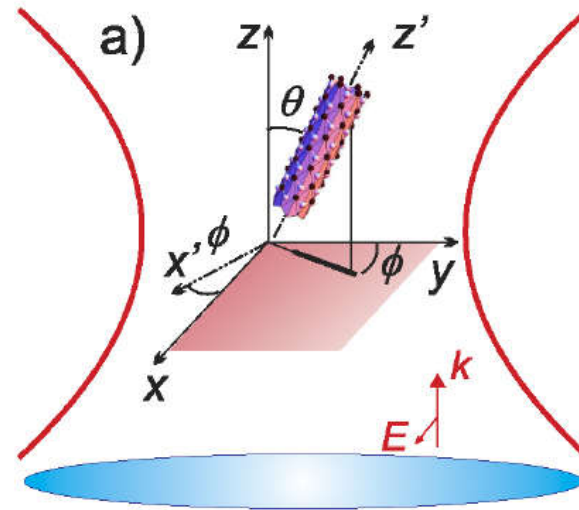
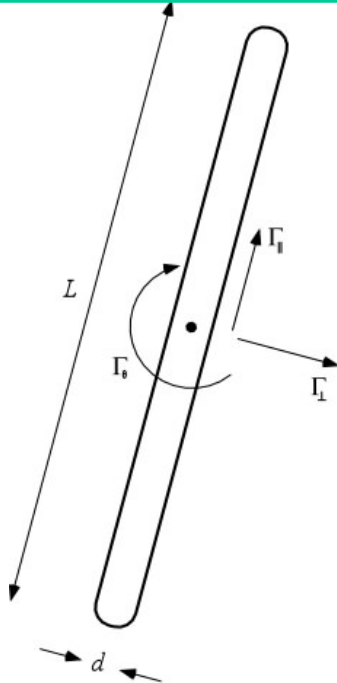


Broersma, J.Chem.Phys. (1981);
Tirado et al. J. Phys Chem C(1984)

Hydrodynamics of a rod-like nanostructure is anisotropic

$$\Gamma_{\perp} = \frac{\ln p + \delta_{\perp}}{4\pi\eta L}, \quad \Gamma_{\parallel} = \frac{\ln p + \delta_{\parallel}}{2\pi\eta L}$$

$$\Gamma_{\Theta} = \frac{3(\ln p + \delta_{\Theta})}{\pi\eta L^3}$$



The signals from the QPD are a composition of center of mass \mathbf{X}_i and angular motion Θ_i .

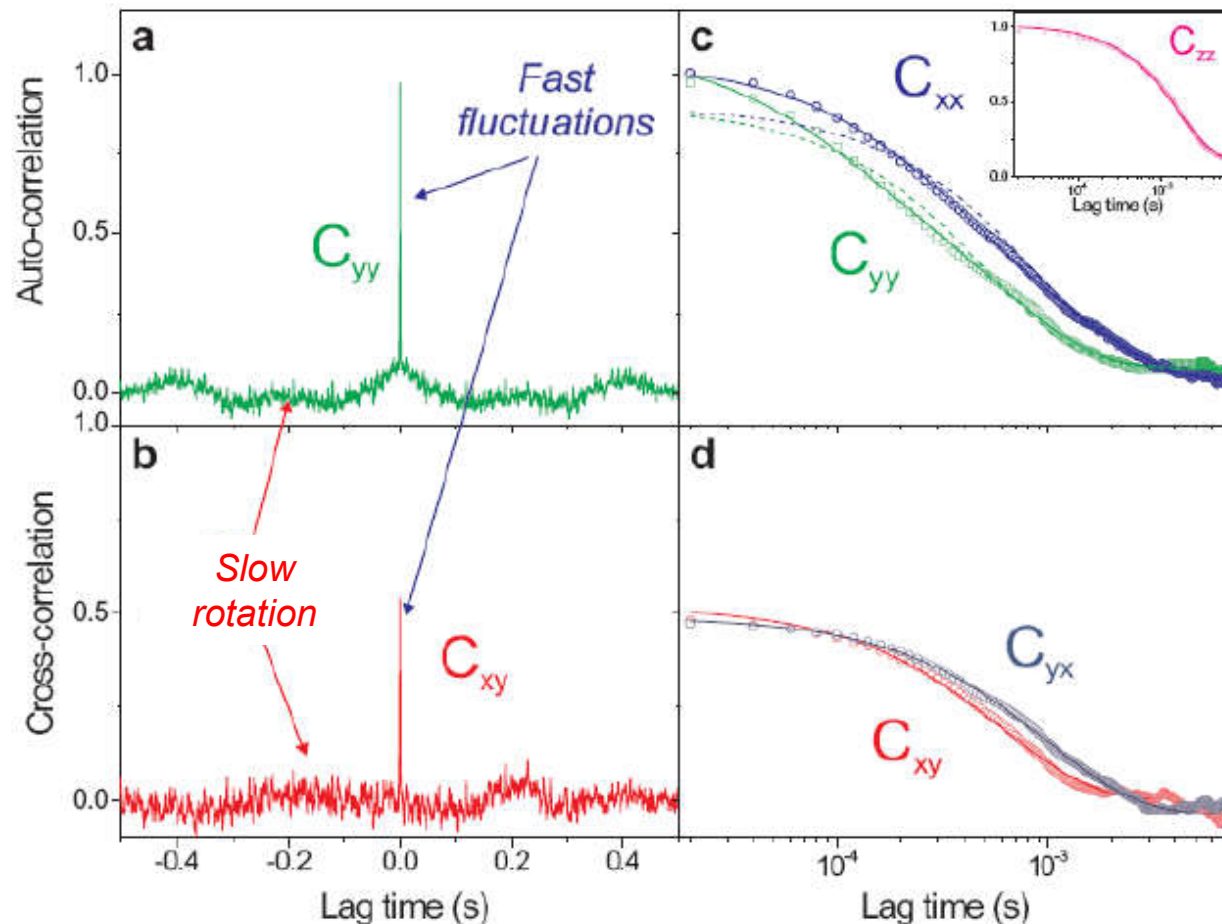
$$S_x \sim \beta_x (X + a \Theta_x); \quad S_y \sim \beta_y (Y + b \Theta_y); \quad S_z \sim \beta_z Z$$

Small angle approximation

Correlation functions of the tracking signals give information on torque and force constants.

$$C_{ii}(\tau) = \langle S_i(t)S_i(t + \tau) \rangle$$

$$C_{xy}(\tau) = \langle S_x(t)S_y(t + \tau) \rangle$$



Double Exp for "fast" dynamics

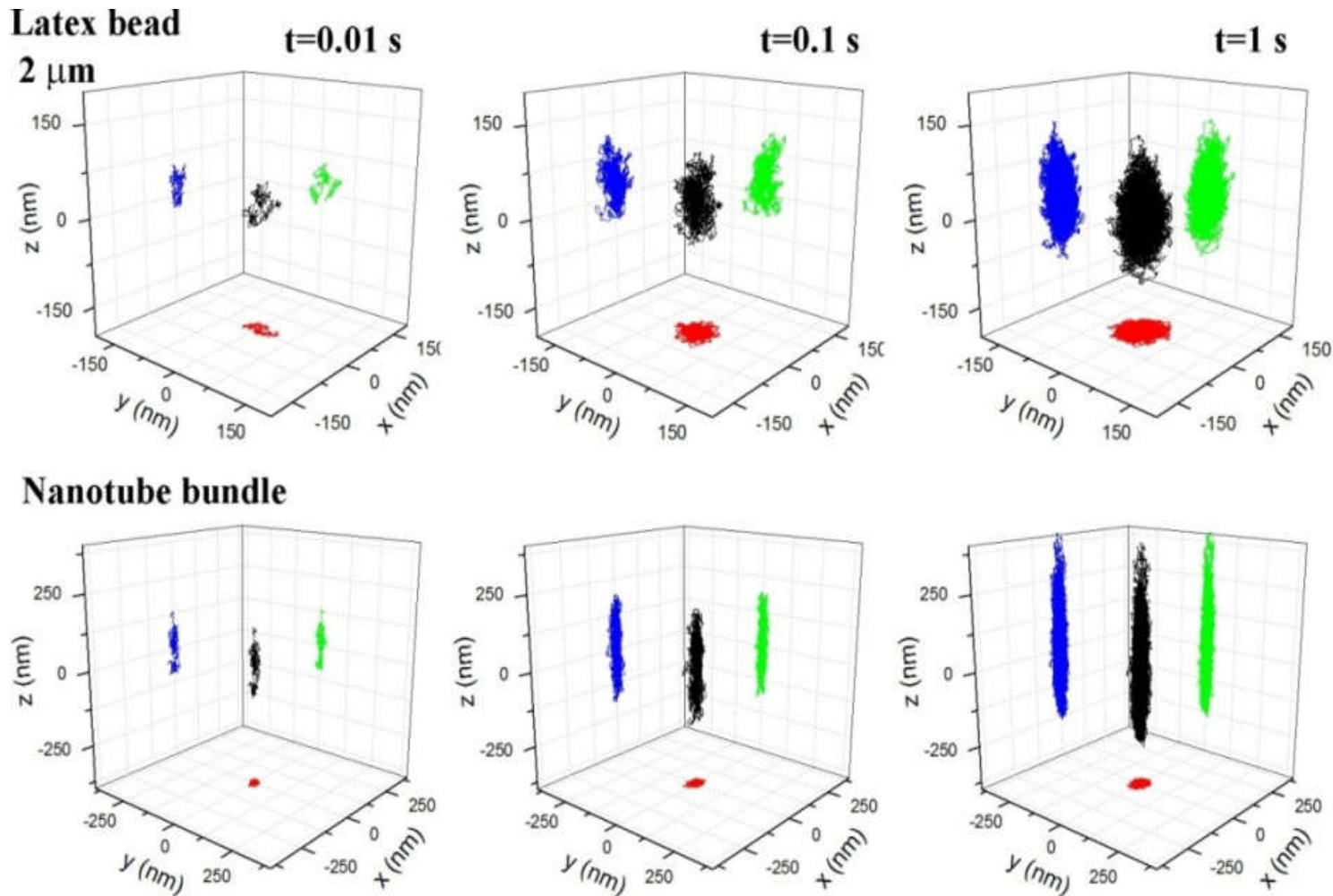
Simple Exp for Z and Cross

Non-conservative forces:
Simpson&Hanna, PR E (2010)



Brownian Motion of Nanotubes/Nanowires

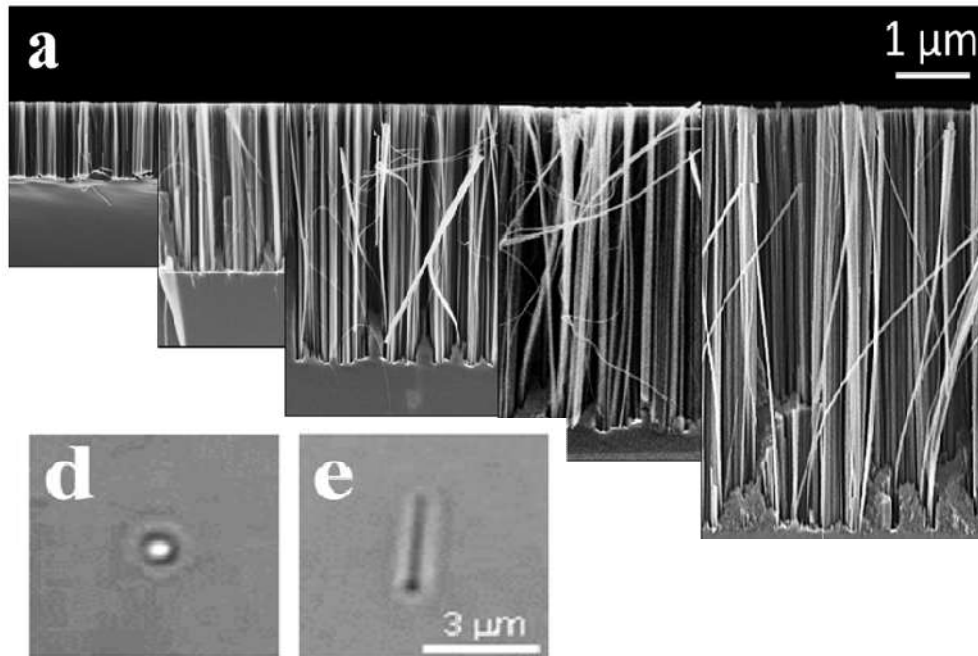
Shape determines the effective potential



Transverse fluctuations in the 10 nm range

A. Irrera et al., *Nano Letters* **2011**, *11*, 4879

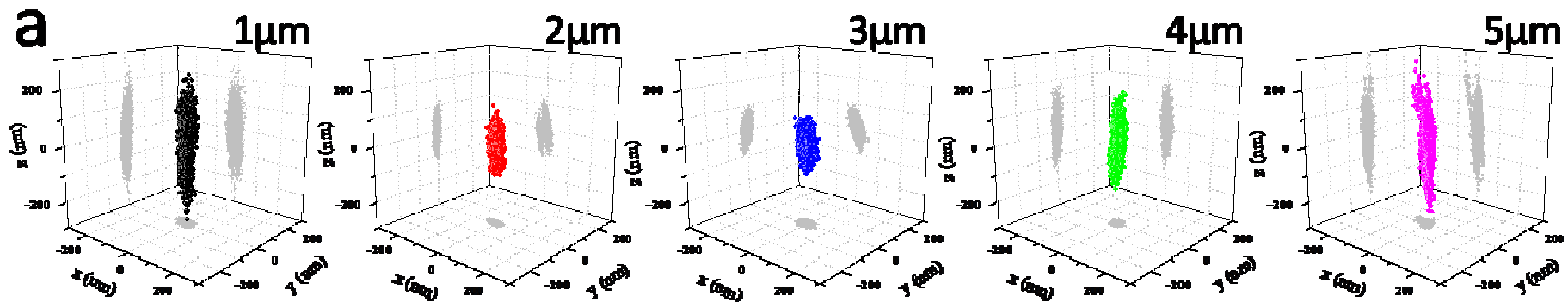
A. Irrera, A. Magazzù, et al., *Nano Letters* **2016**.



We can now control the length and the diameter

Length controls optical forces and torques

Size-scaling with the size parameter $x_L = \pi n L / \lambda$



Optical trapping of SiNW with controlled size

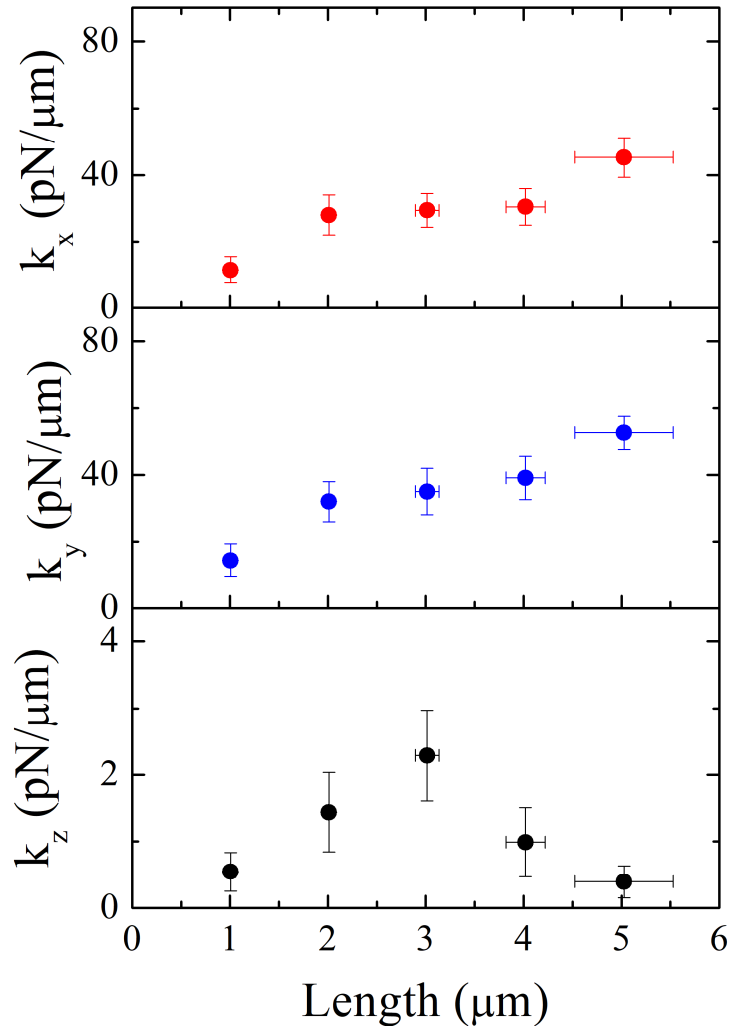


Scaling of Force Constants



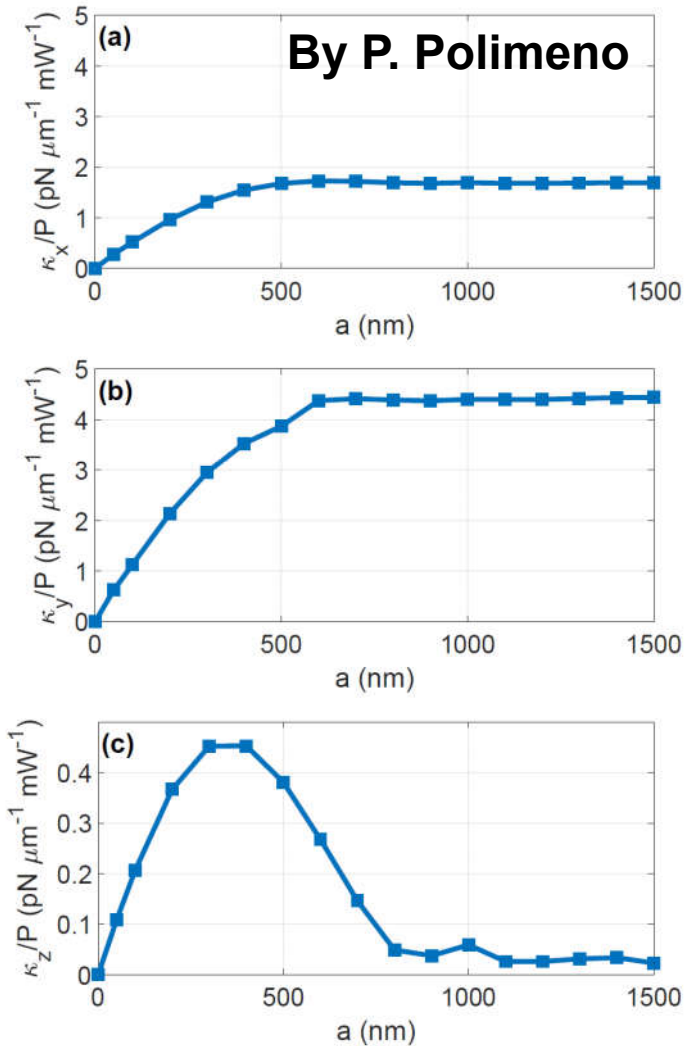
Experiment

Irrera et al., Nano Letters (2011)

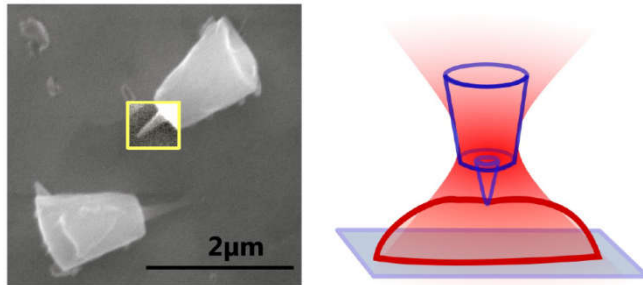


For long nanowires transverse force saturates, axial force drops.

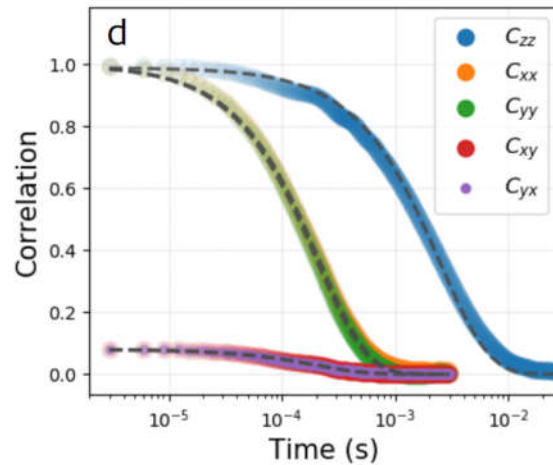
T-matrix



Nanofabricated birifrengent particles with nanometric needles

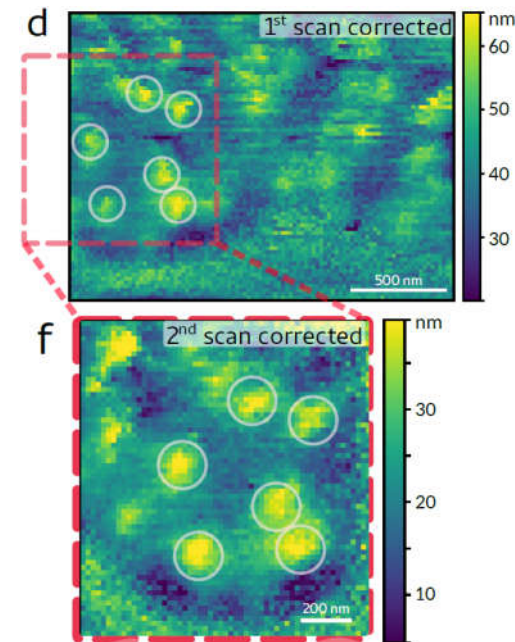
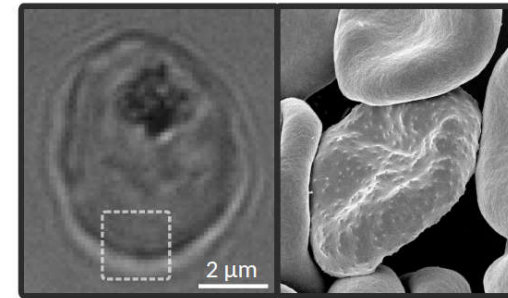


Accurate force calibration of non-spherical probes



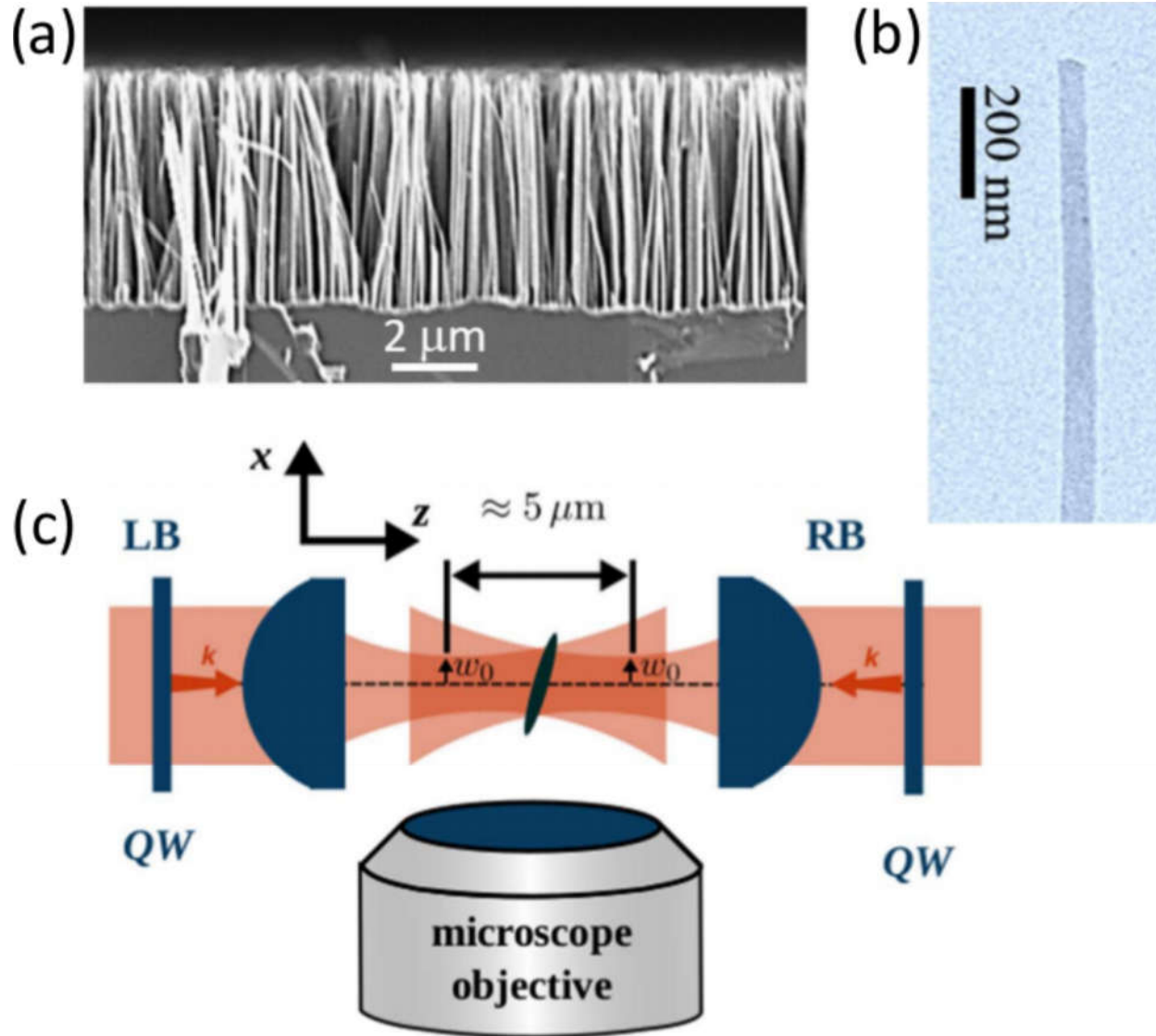
Topography of soft materials

Membrane of living malaria-infected red blood cells presenting knobs





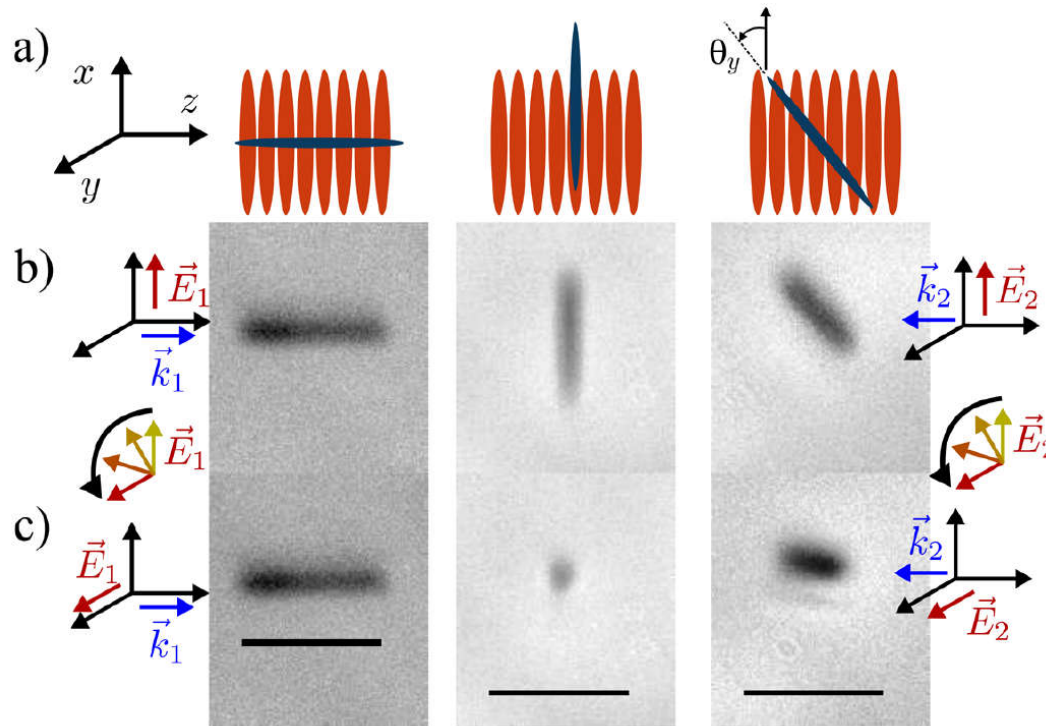
Silicon nanowires in 2-beam traps



Donato, Brzobohaty, Simpson, et al. Nano Letters 19, 342 (2019)

Silicon nanowires in 2-beam traps

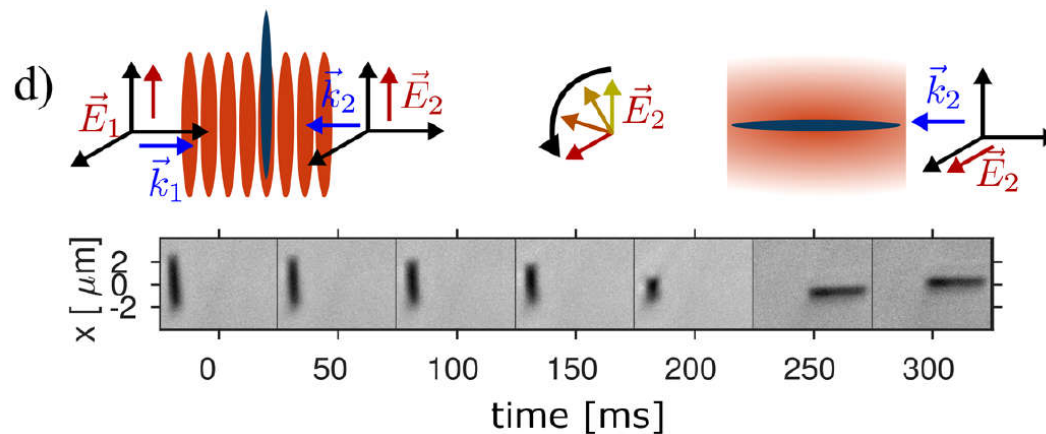
Rich scenario regulated by shape & polarization



Parallel linear polarization (PLP) leads to **several equilibrium configuration** regulated by the **length** of the NWs

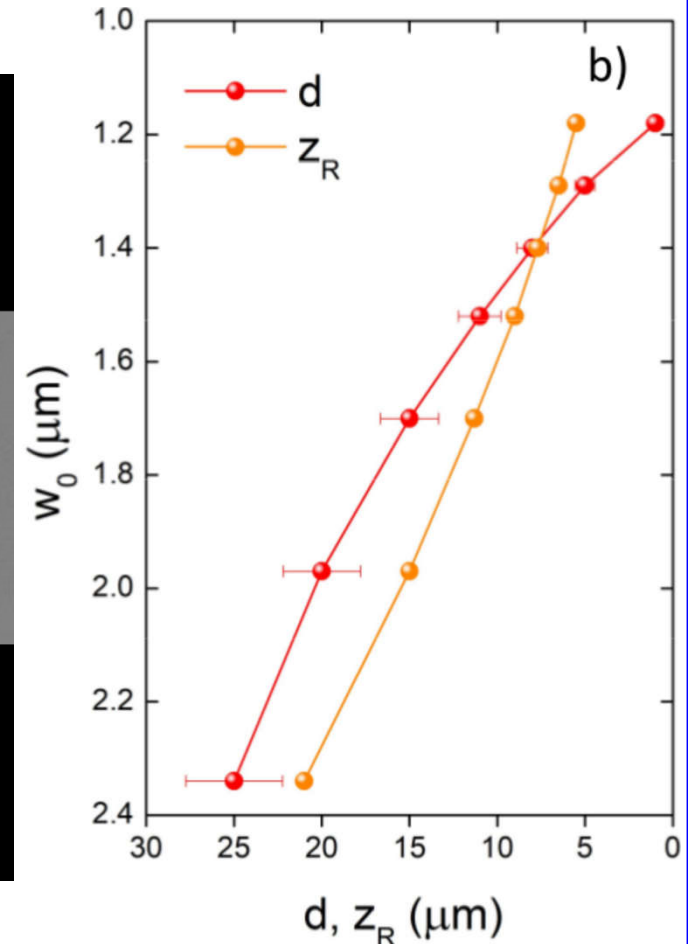
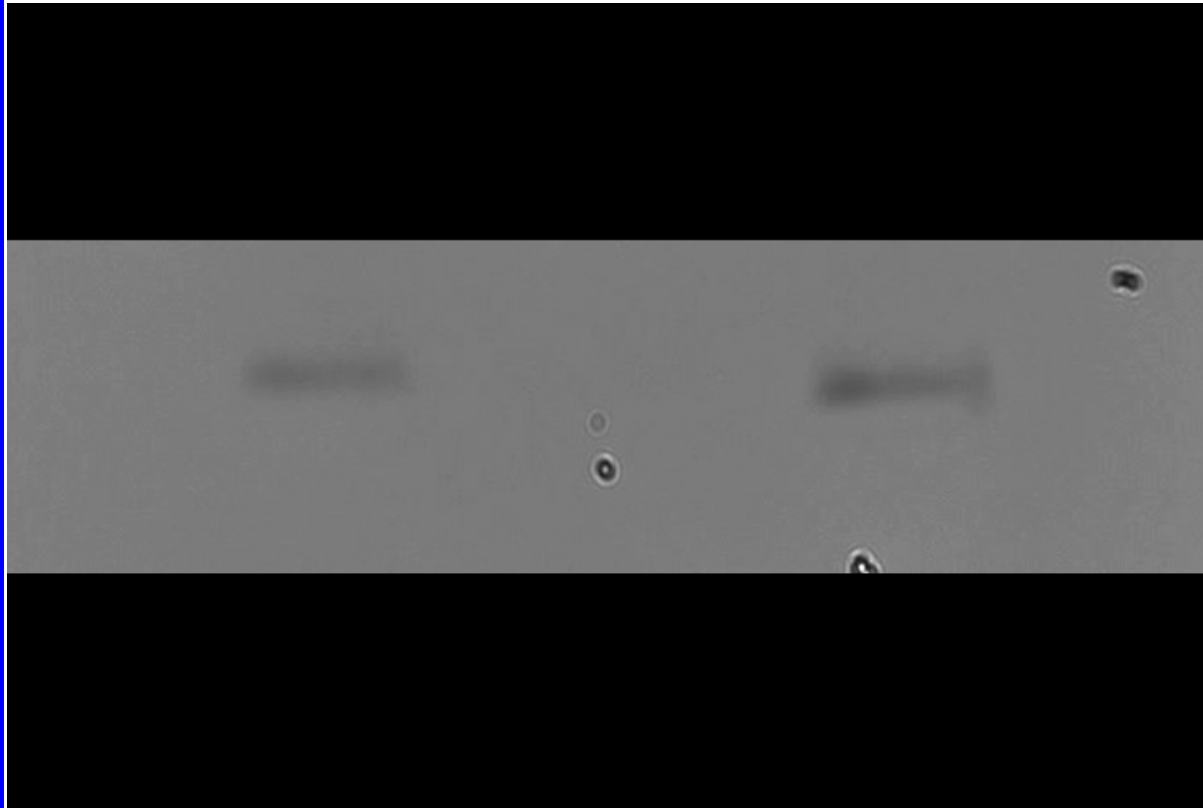
The NW scatters "mainly" from end-tips

Polarization can be used to control orientation



From PLP to XLP (fringes disappear) longitudinal torque wins over polarization torque

Optical binding of nanowires in cross linear polarization



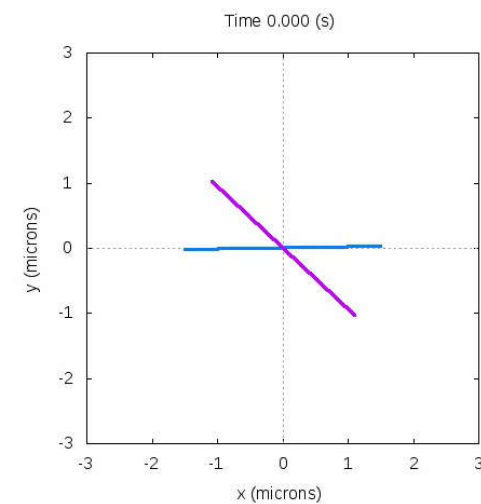
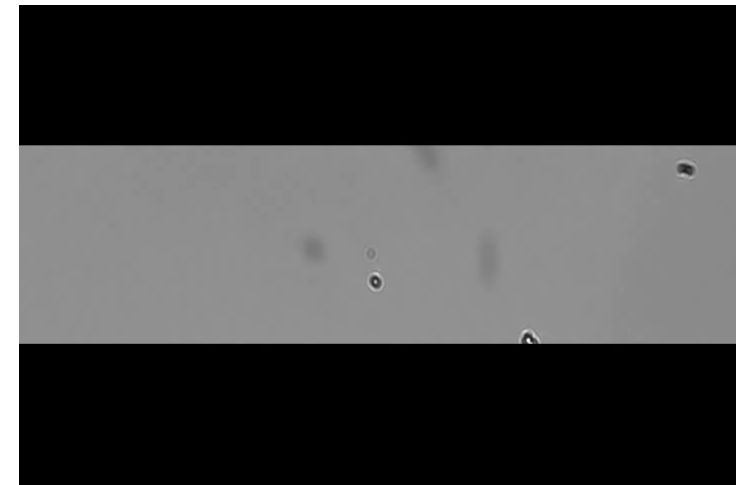
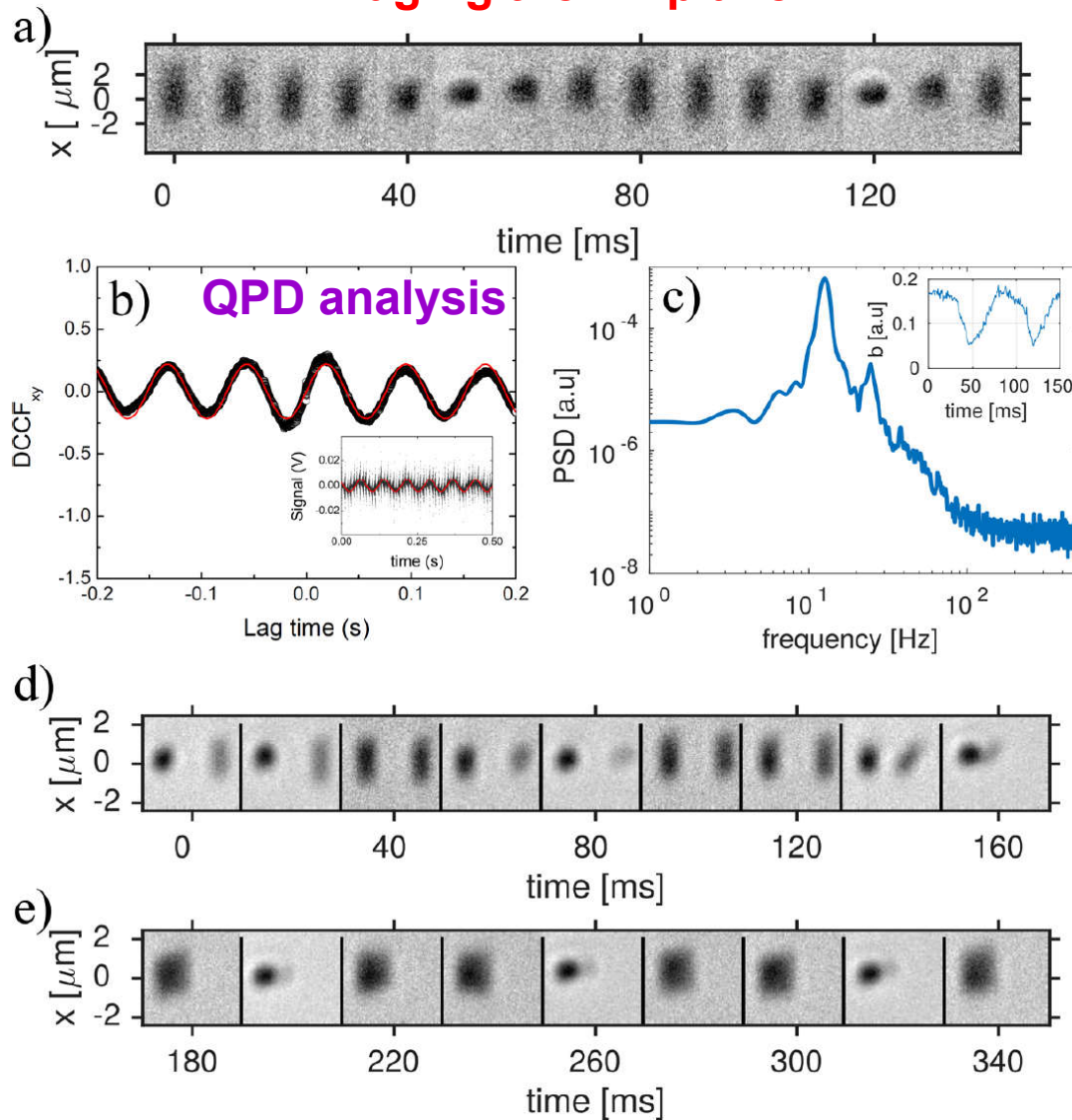
By changing the size, w_0 , of the laser beams we can control the distance and binding interaction



Rich spin rotational dynamics regulated by shape



Imaging the XZ plane

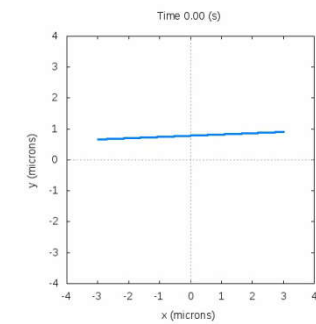
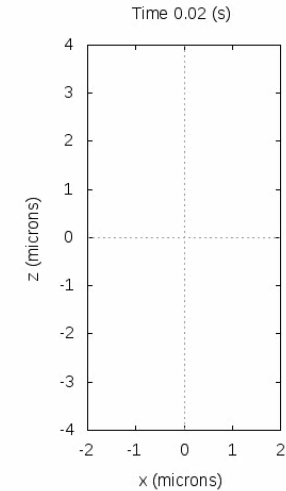
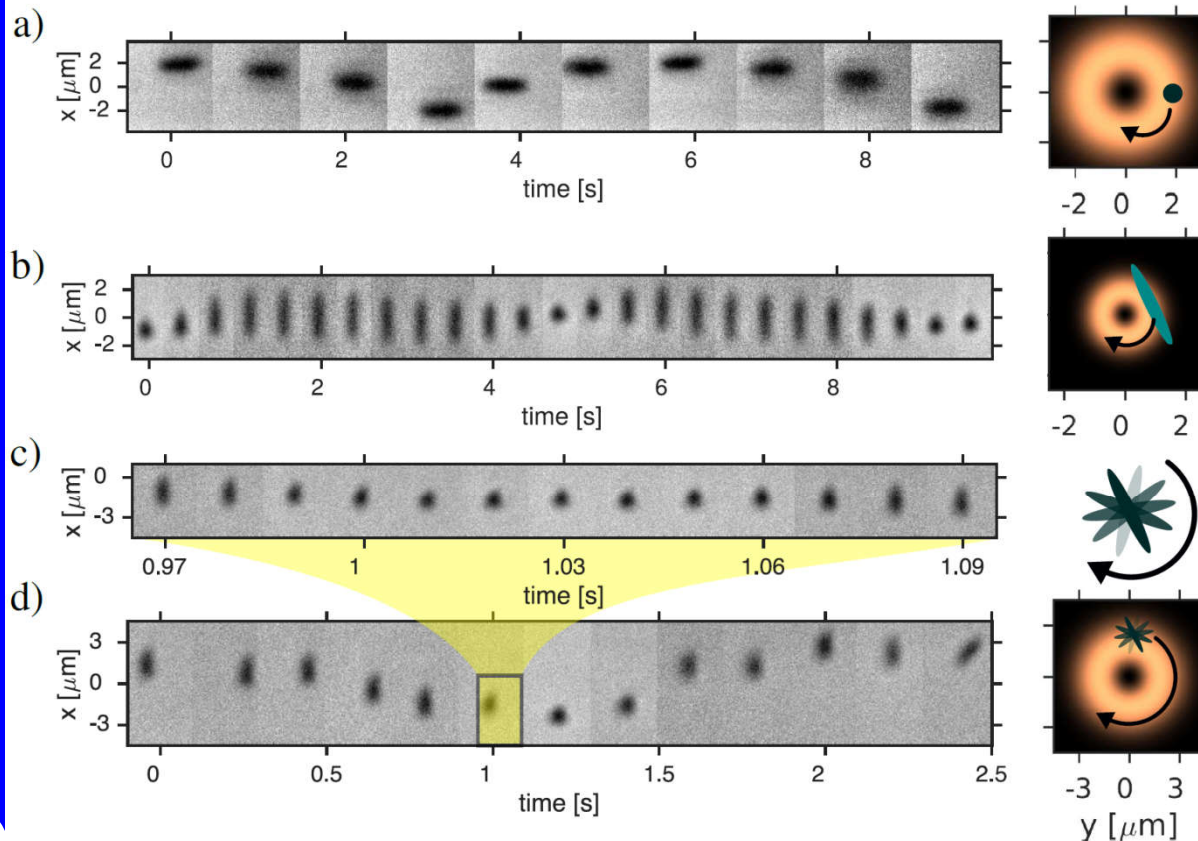
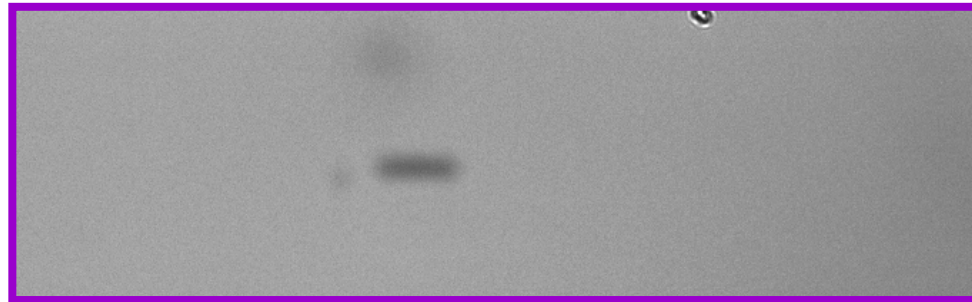


Simulations by S. H. Simpson

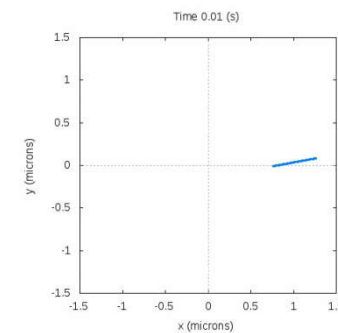
Rich orbital rotational dynamics regulated by shape

Imaging the XZ plane

Sim. by S. H. Simpson **XZ**



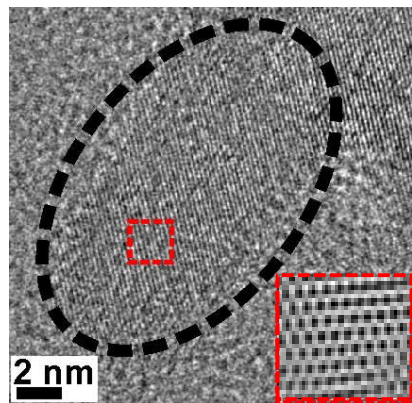
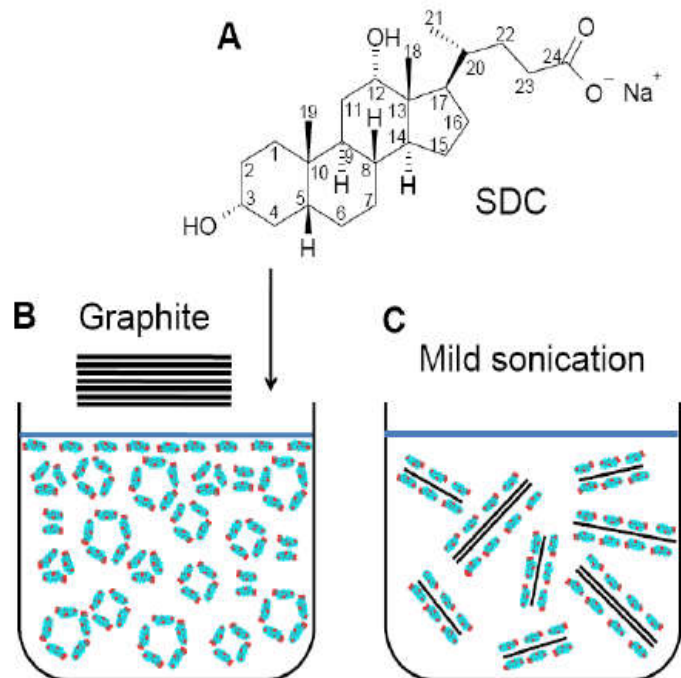
XY



XY

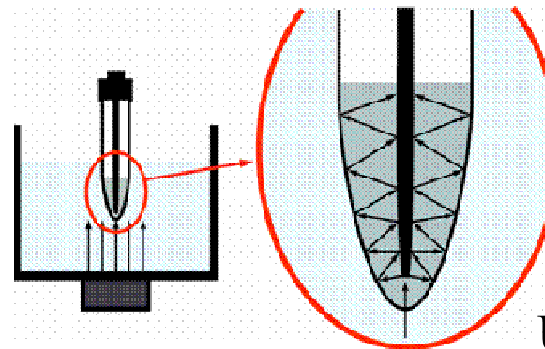
Layered materials

Graphene

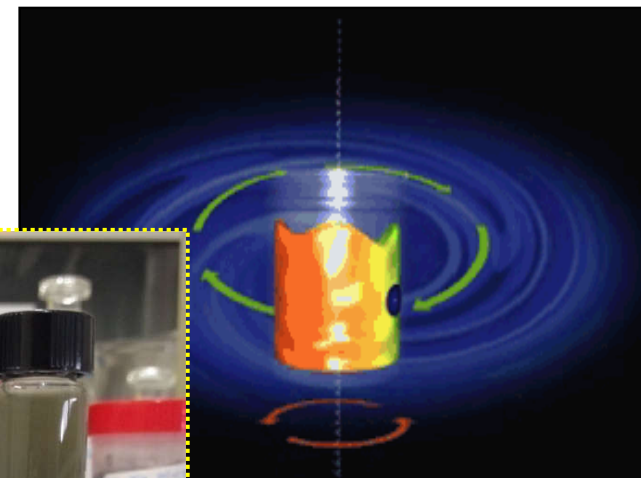


2d Materials

Ultrasonication



Ultracentrifugation

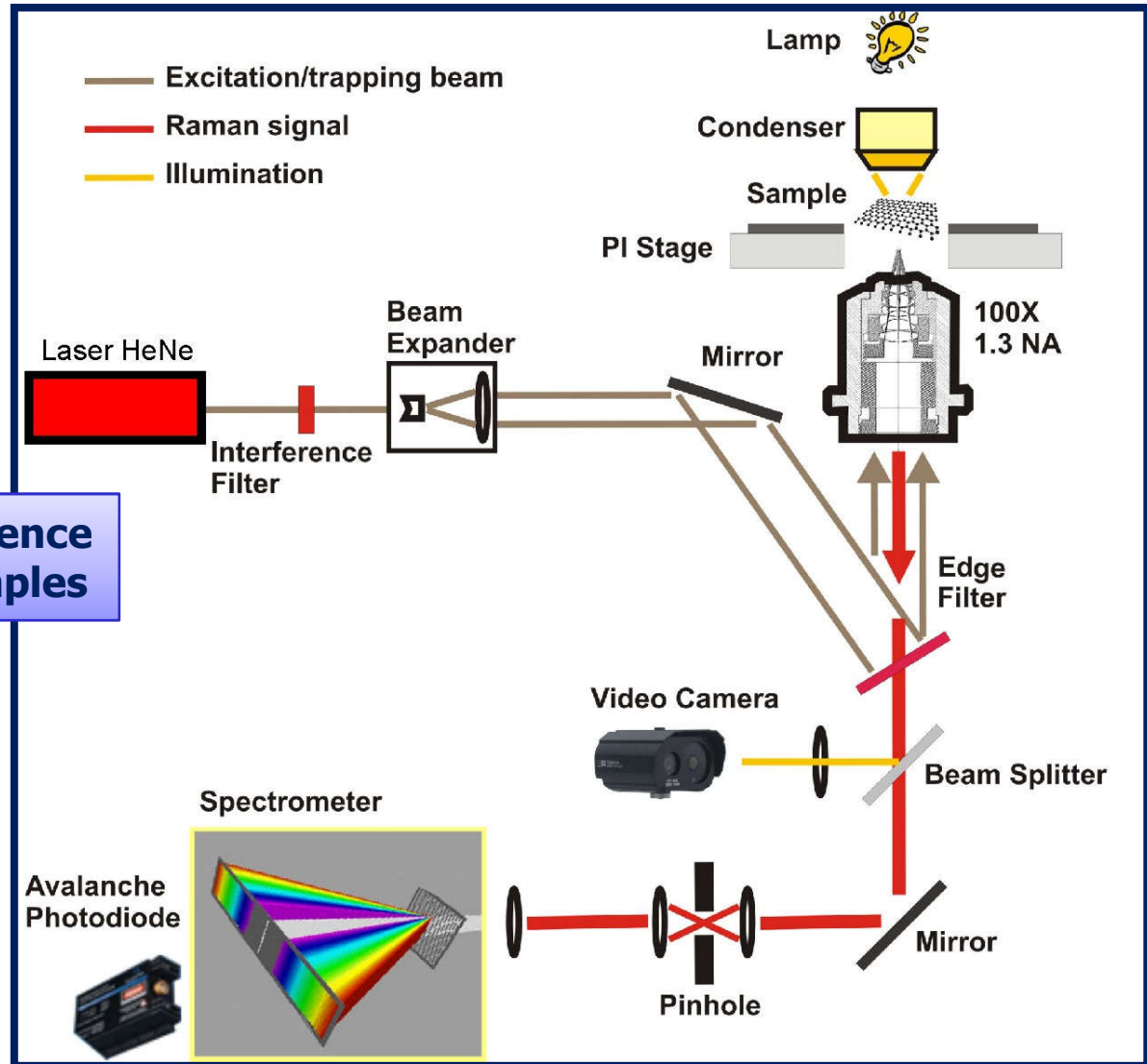
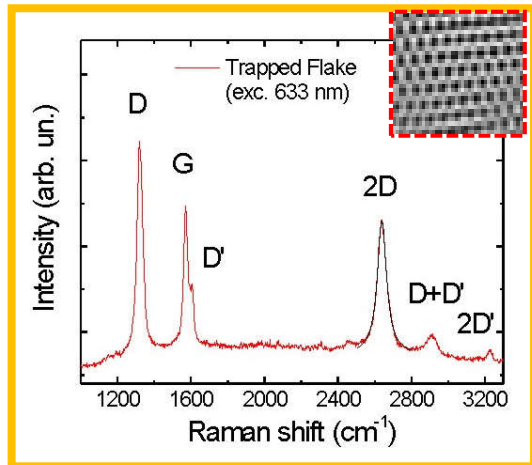


2d Materials Inks
Flexible electronics
and textile

- 561nm, 633nm, 785nm...
- Edge/Notch filter
- Jobin-Yvon Triax 190 spectrometer
- Avalanche photodiode, SPC (Perkin Elmer)

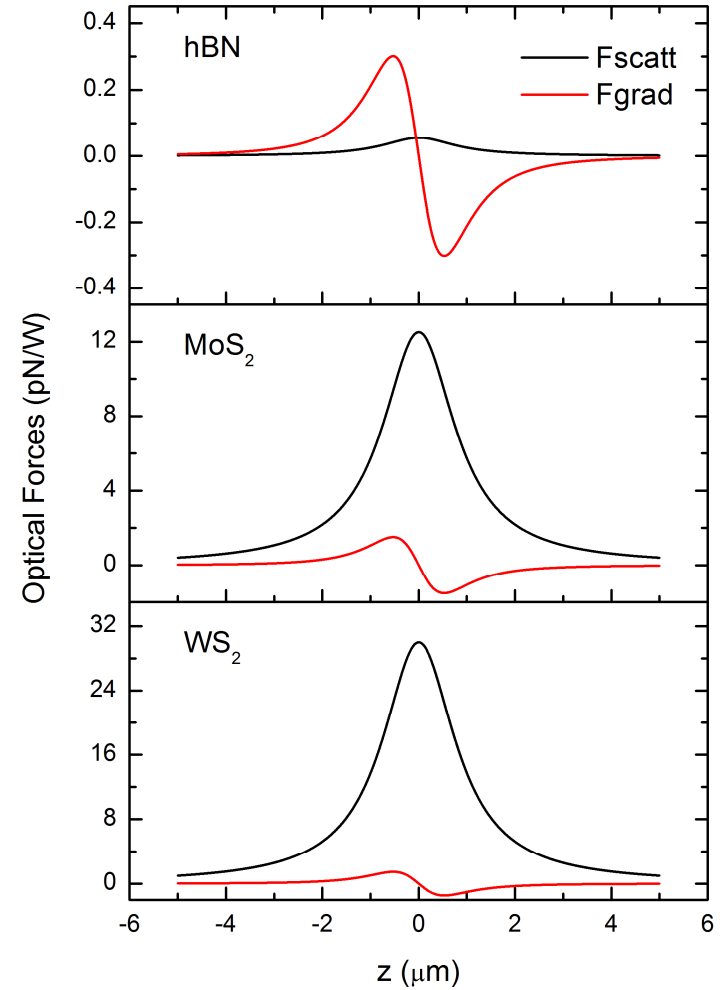
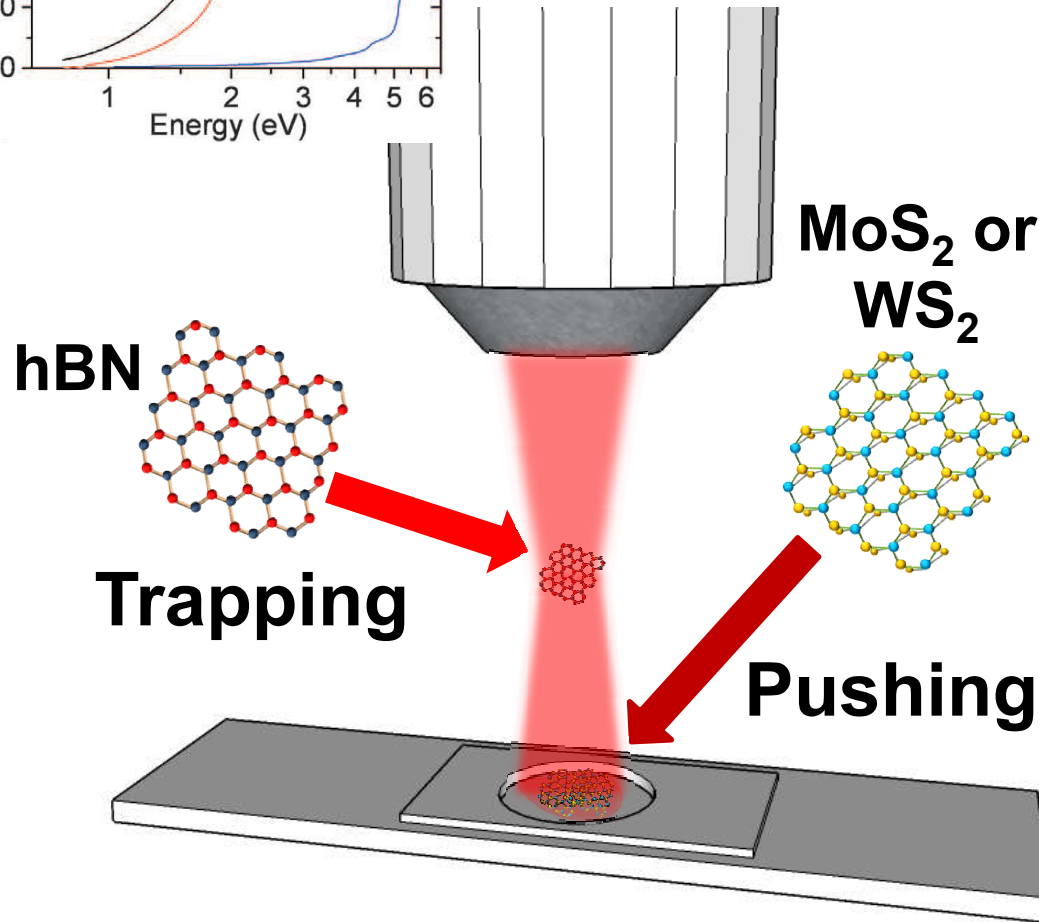
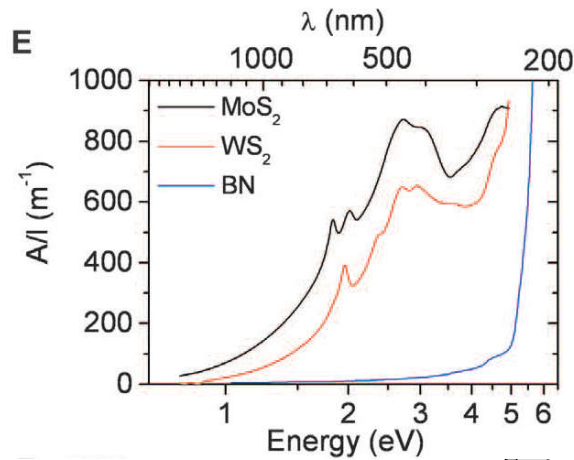
Raman & Photoluminescence inspection of trapped samples

E.g., Inspection of **Graphene** flakes

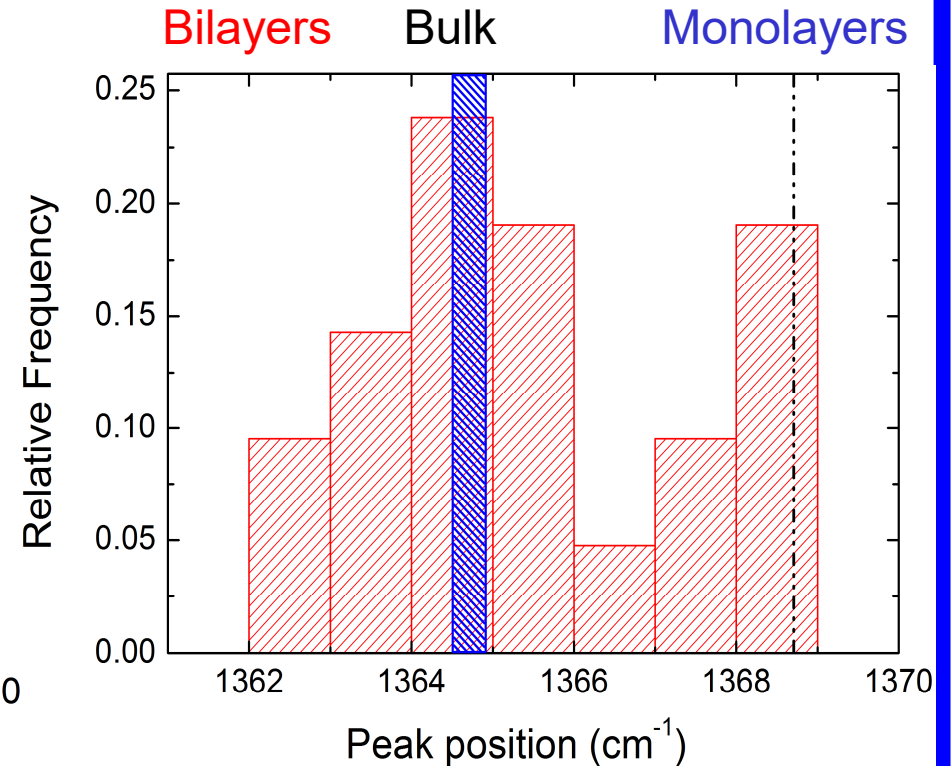
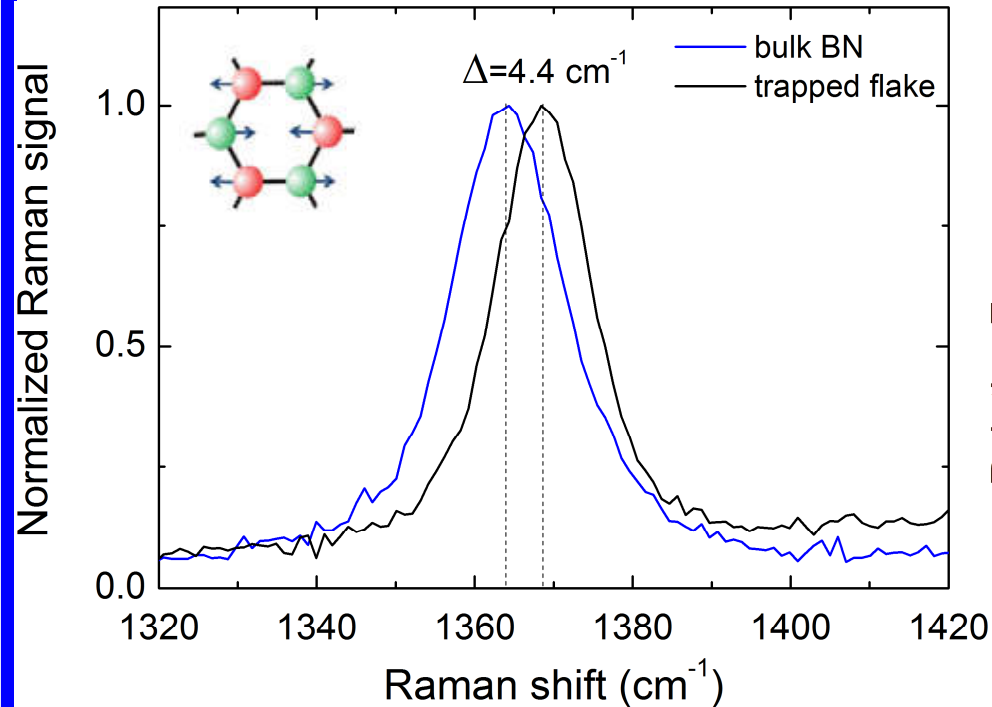


Maragò, et al., "Brownian Motion of Graphene", *ACS Nano* **4**, 7515 (2010)
 Maragò, et al. *Nature Nanotechnology* **8**, 807–819 (2013)

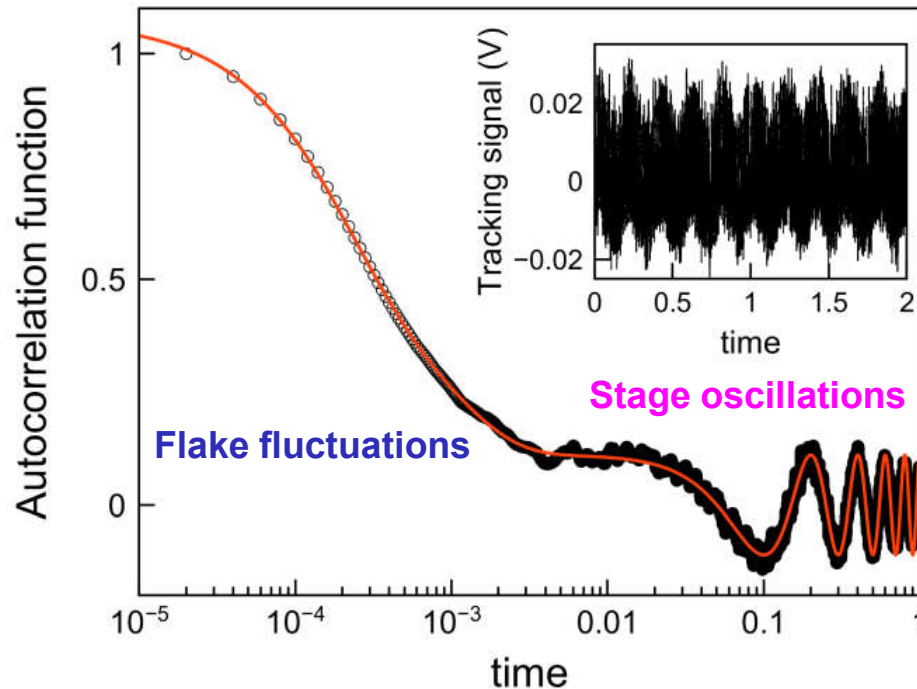
Mechanical effects of light on layered materials



Donato et al., *Nanoscale* (2018)



- hBN flakes trapped with **785 & 830 nm laser**
- Trapped flakes have a range of peak positions. The blue-shifted positions (4cm^{-1}) of some peaks ($\sim 1370\text{cm}^{-1}$) suggests that **monolayer flakes** are present in the sample. Presence of **bi-layers** (that should show a 2 cm^{-1} red shift) is also evident.



Microscope stage calibration combined with ACFs

Microscope **stage oscillation** yield a sinusoidal modulation of the particle's displacement

$$V_x(t) = x_V(t) + a_V \sin(\omega_{stage} t)$$

All parameters from ACF fitting

Hydrodynamics for a thin disk

$$\begin{aligned} \gamma_{\parallel} &= 8\eta D \\ \gamma_{\perp} &= \frac{16}{3}\eta D \\ \gamma^r &= \frac{4}{3}\eta D^3 \end{aligned}$$

$$C_{xx}^V(\tau) = Ae^{-\omega_x \tau} + \frac{a_V^2}{2} \cos(\omega_{stage} \tau) = \beta_x^2 \frac{k_B T}{k_x} e^{-\omega_x \tau} + \frac{a_V^2}{2} \cos(\omega_{stage} \tau)$$

ACFs full calibration enable flake size estimates from a simple hydrodynamic model

Donato et al., Nanoscale (2018)



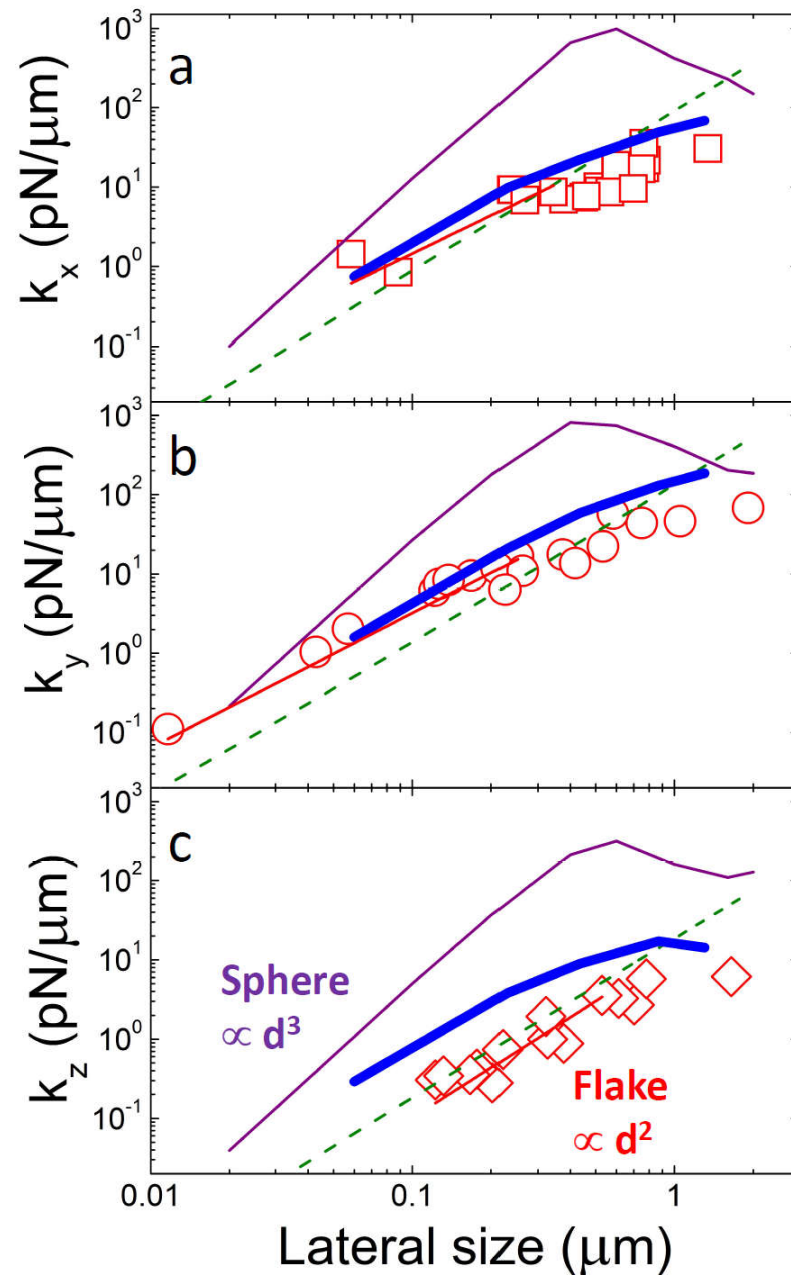
Optical force scaling on hBN



Two-dimensional scaling of optical trapping forces

Polarizability depends on the **area** of the layered particle

Flattening for large flakes (dipole approximation breaks down)

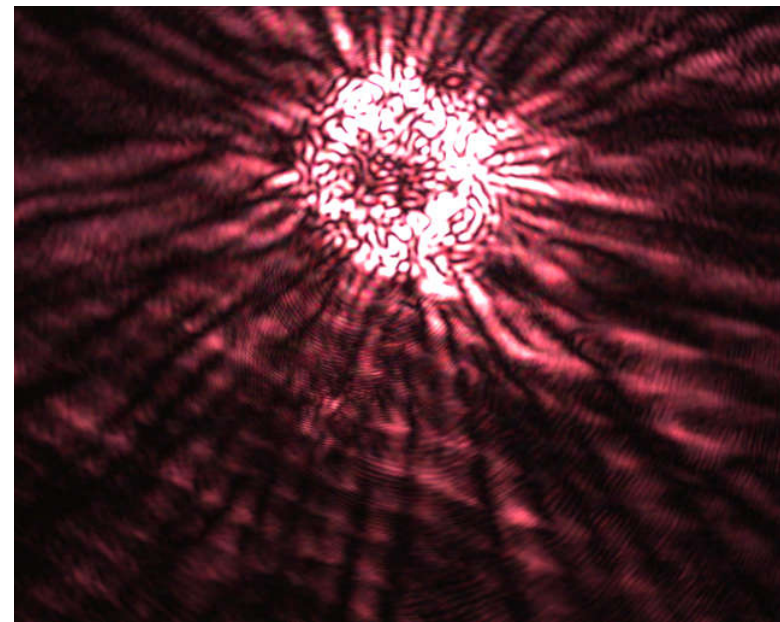
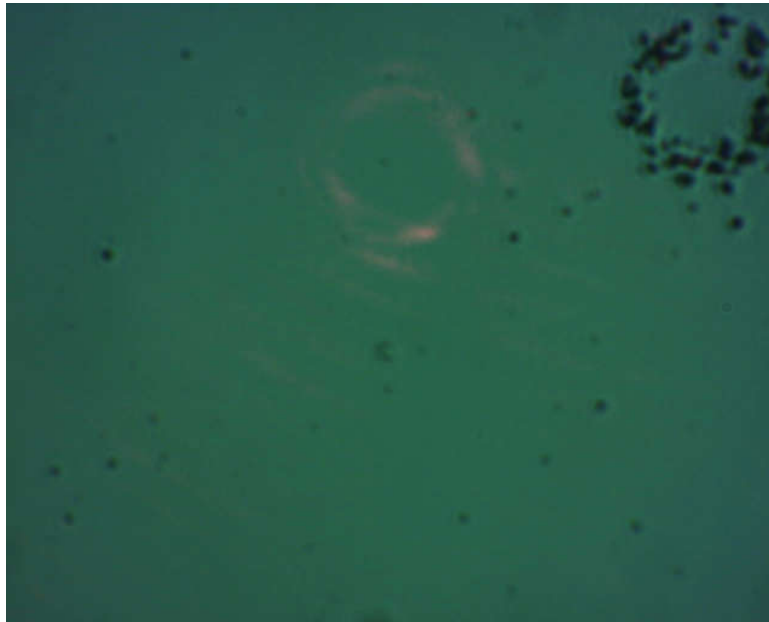


Optical force positioning of MoS₂ & WS₂

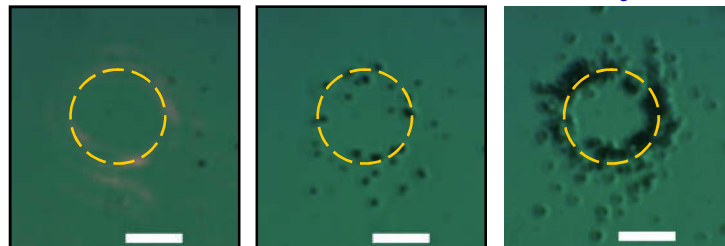
Begin pushing

LG-Beam ($l=30$)

After few minutes

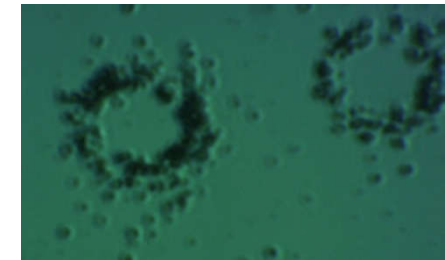
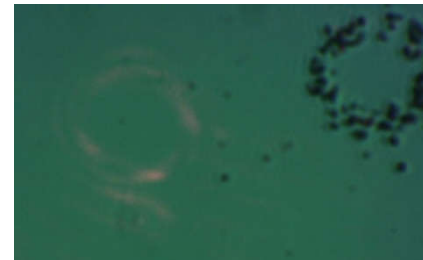


time



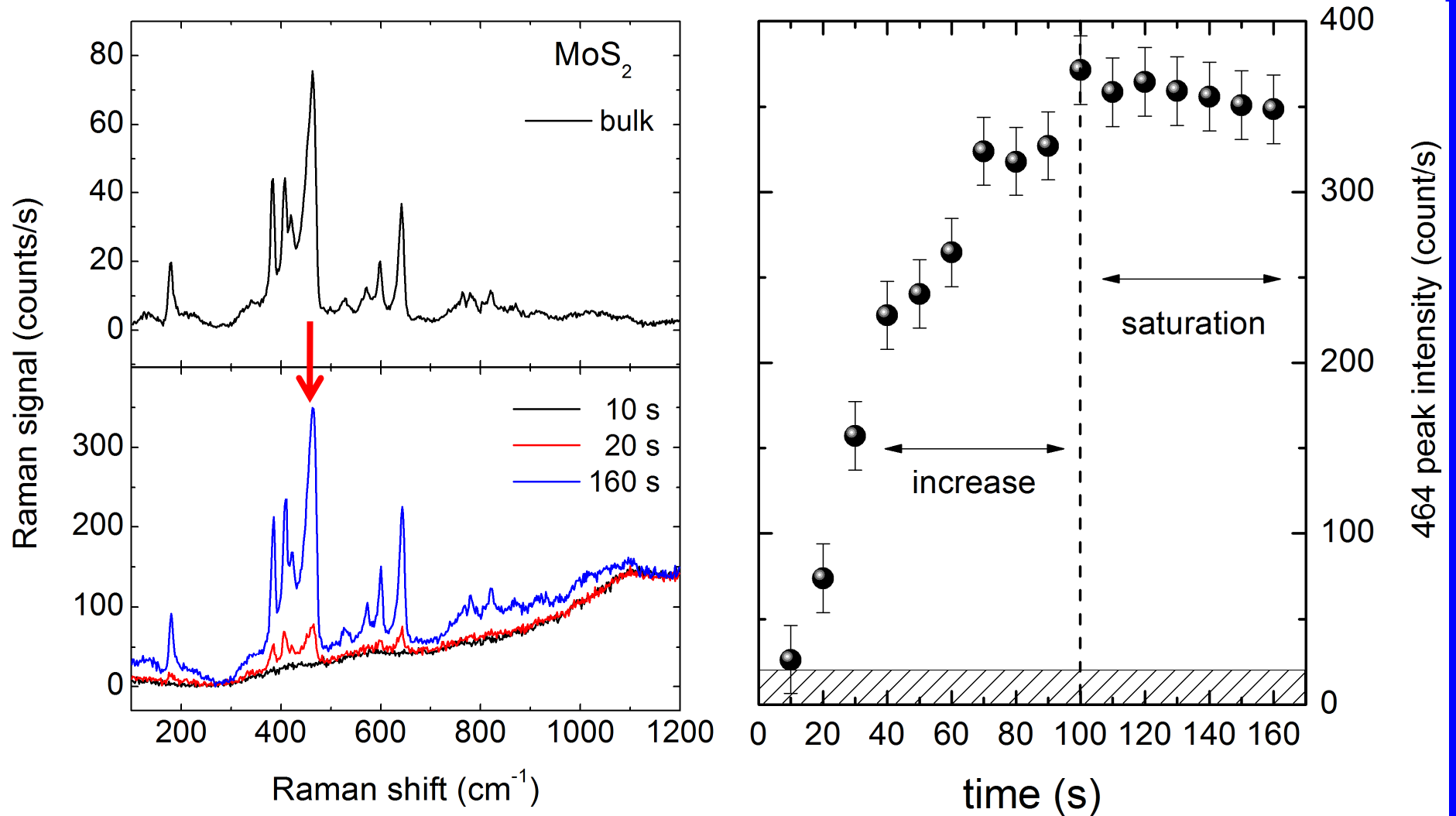
start

end

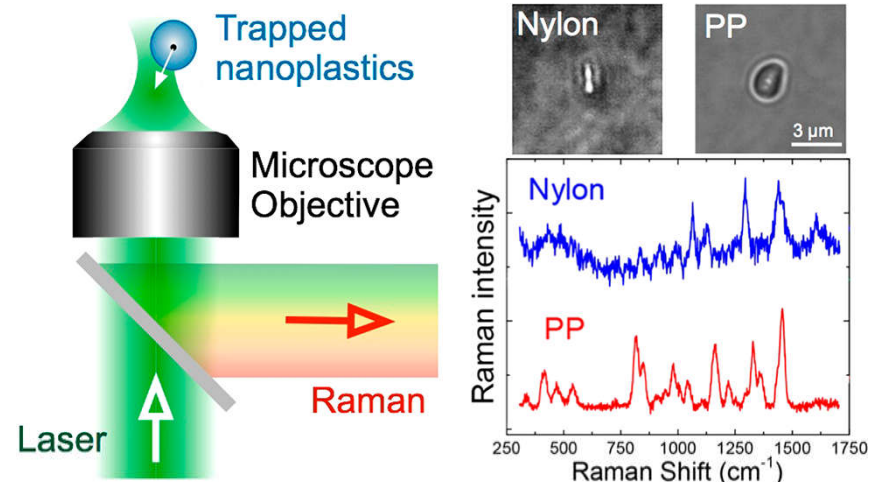
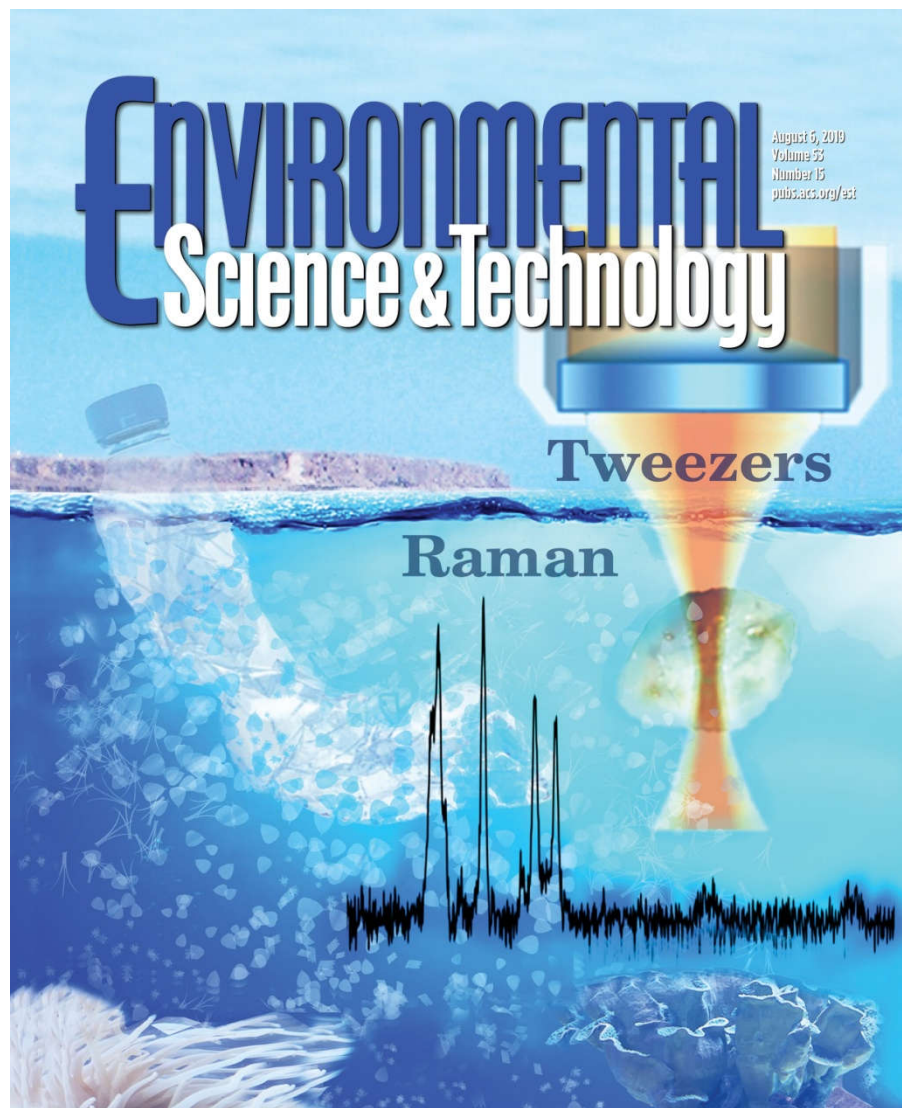


Use proteins (BSA) to glue the structures to a simple glass substrate

Time-evolution of optical force aggregation of MoS₂



Donato et al., *Nanoscale* (2018)



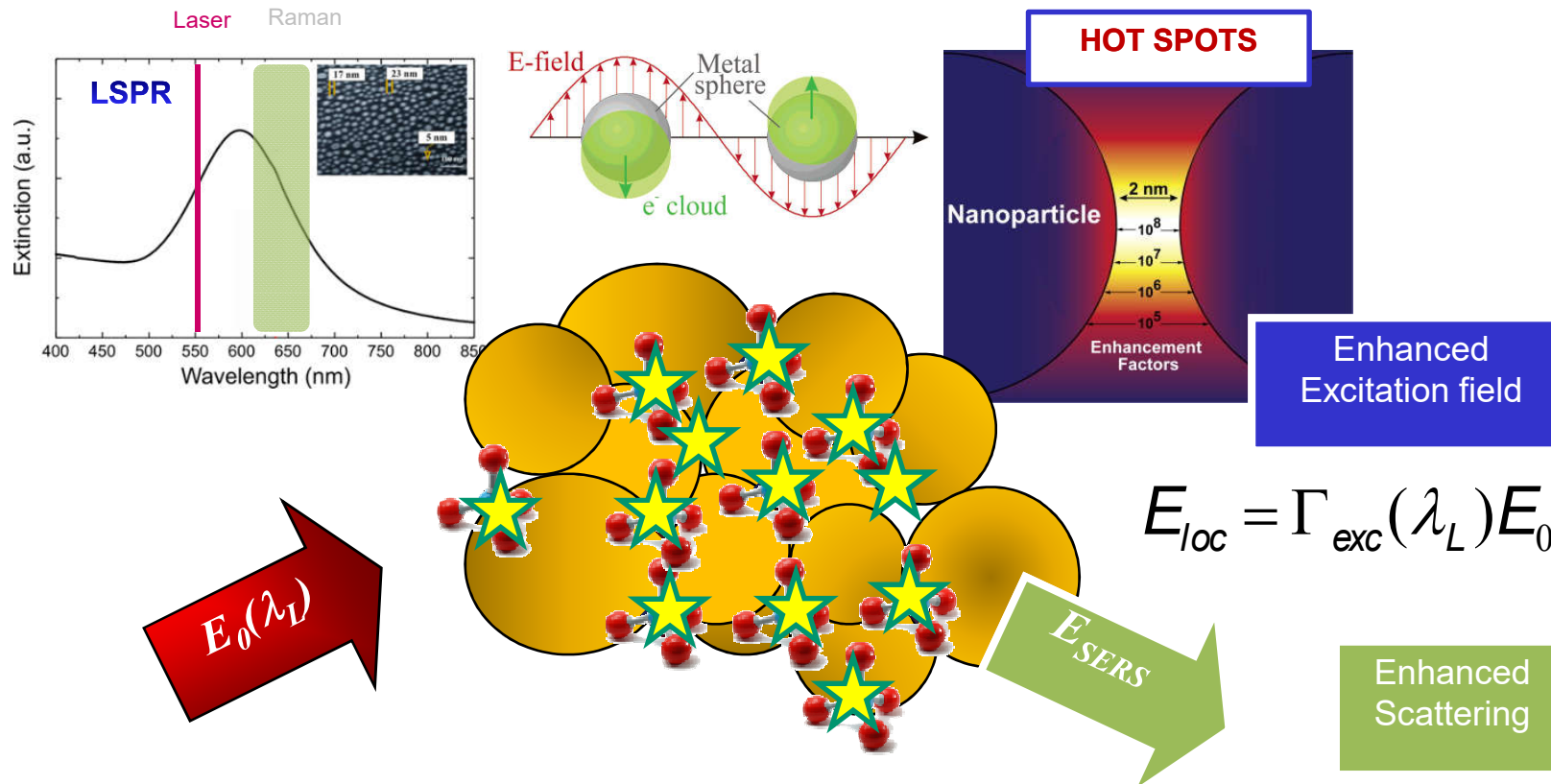
Raman Tweezers permits one to assess the **size and shape** of particles (beads, fragments, and fibers), with spatial resolution only limited by diffraction

Gillibert, R., et al. *Env. Science & Technol.* **53** (2019): 9003.



SERS Tweezers

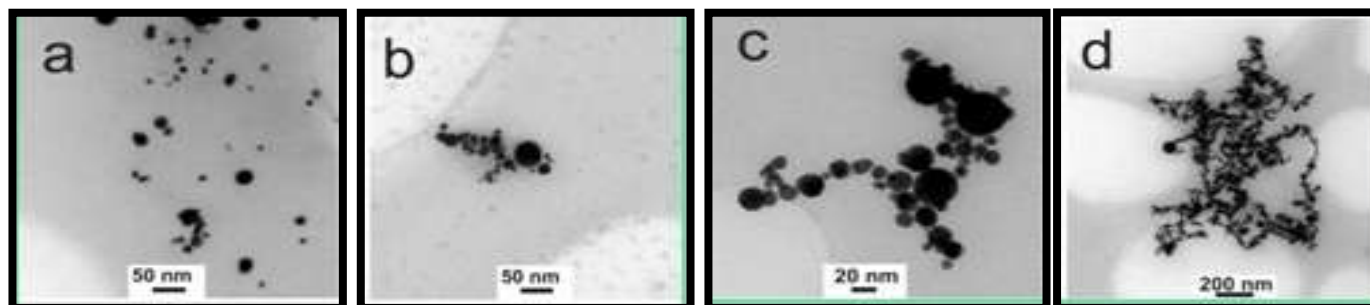
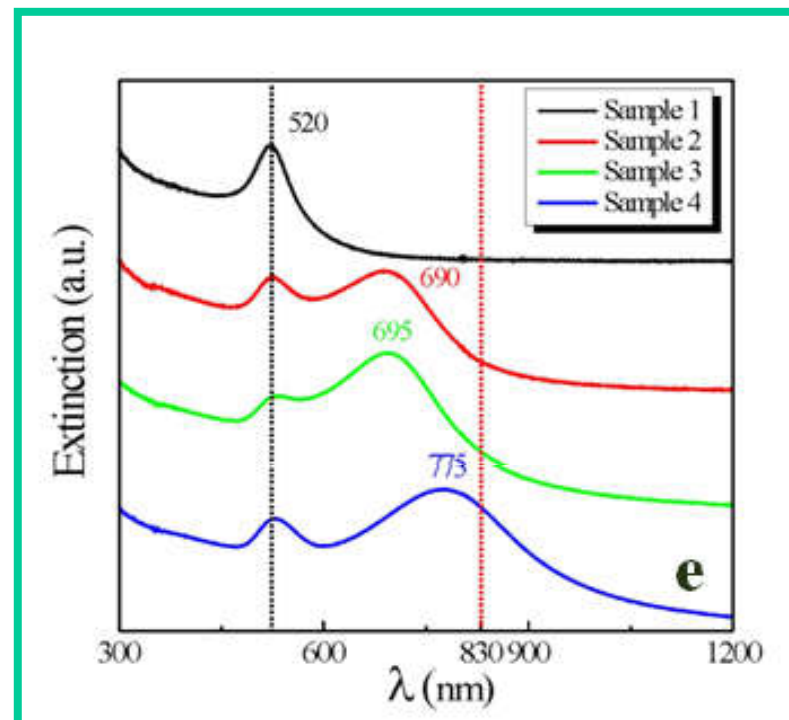
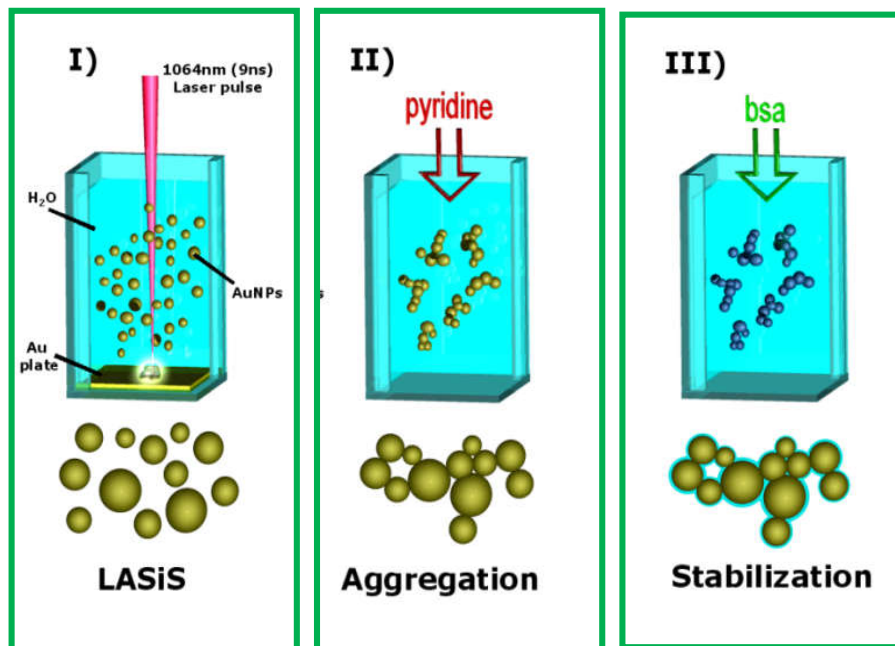
1. Molecules laying on metal nanoparticles and in their interstices experience an enhanced excitation field due to localized plasmon resonances
2. Molecules will experience an enhancement of the scattered fields



➤ Fleischman et al., *Chem Phys Lett*, 26 (1974), p. 123
 ➤ Albrecht MG, Creighton JA, *J. Am. Chem Soc.*, 99 (1977), p. 5215
 ➤ Jeanmaire DL and Duayne RPV, *J. Electroanal. Chem*, 84 (1977), p.1
 ➤ E.C. Le Ru, P.G. Etchegoin, *Phys. Chem. Chem. Phys.*, 10 (2008), p. 6079



LASiS





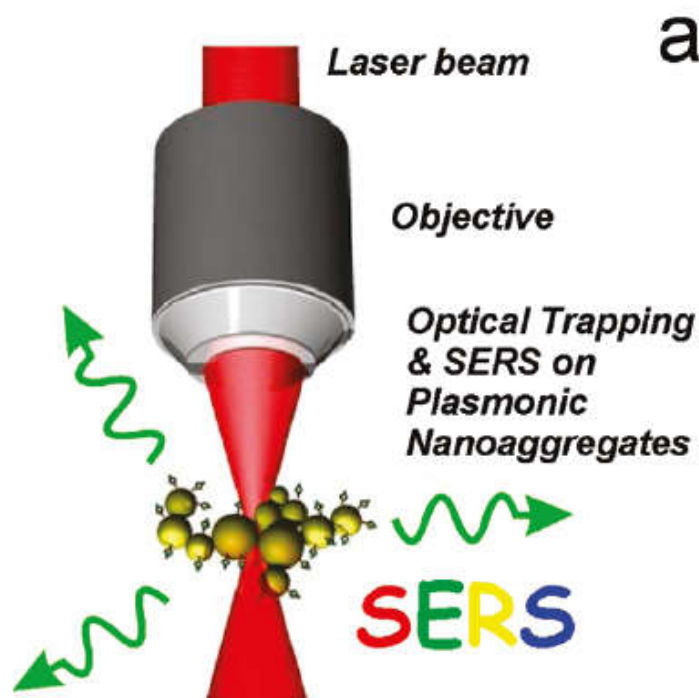
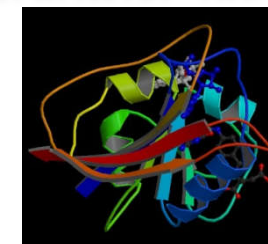
Surface-Enhanced Raman Tweezers



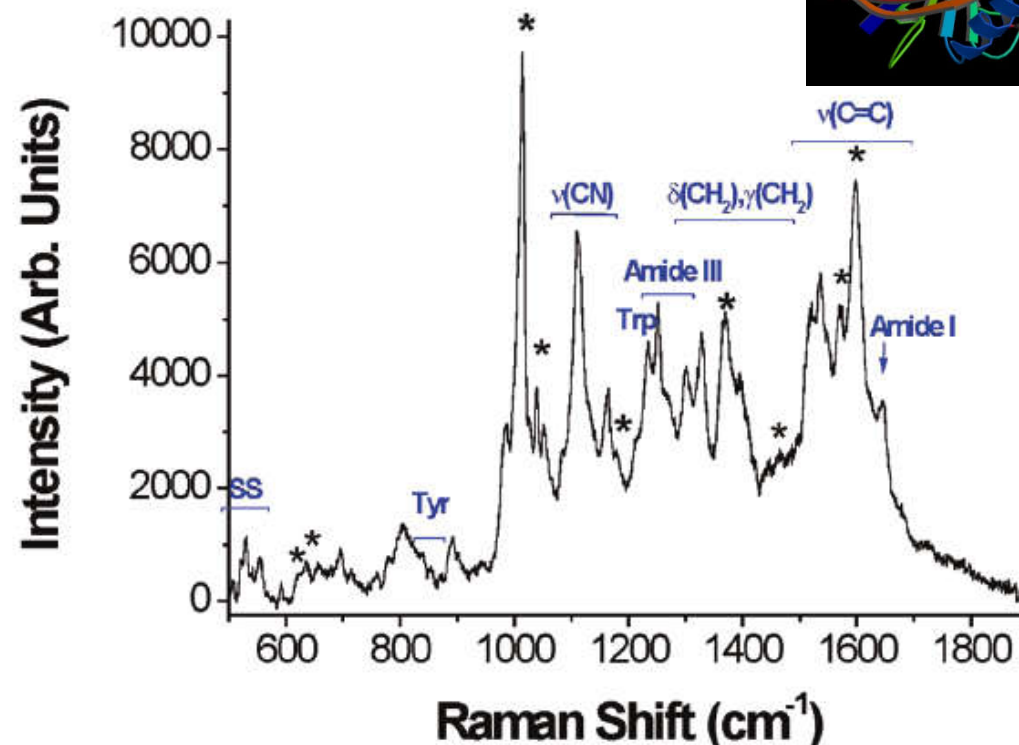
V. Amendola

Use the SAME light to trap and excite SERS
Trapping wavelength 785nm > 695nm MNP Aggregate SPR

BOVINE SERUM ALBUMIN (BSA)



a



Messina et al., *J Phys Chem C* **115**, 5115 (2011)
Messina et al., *ACS Nano* **5**, 905 (2011)

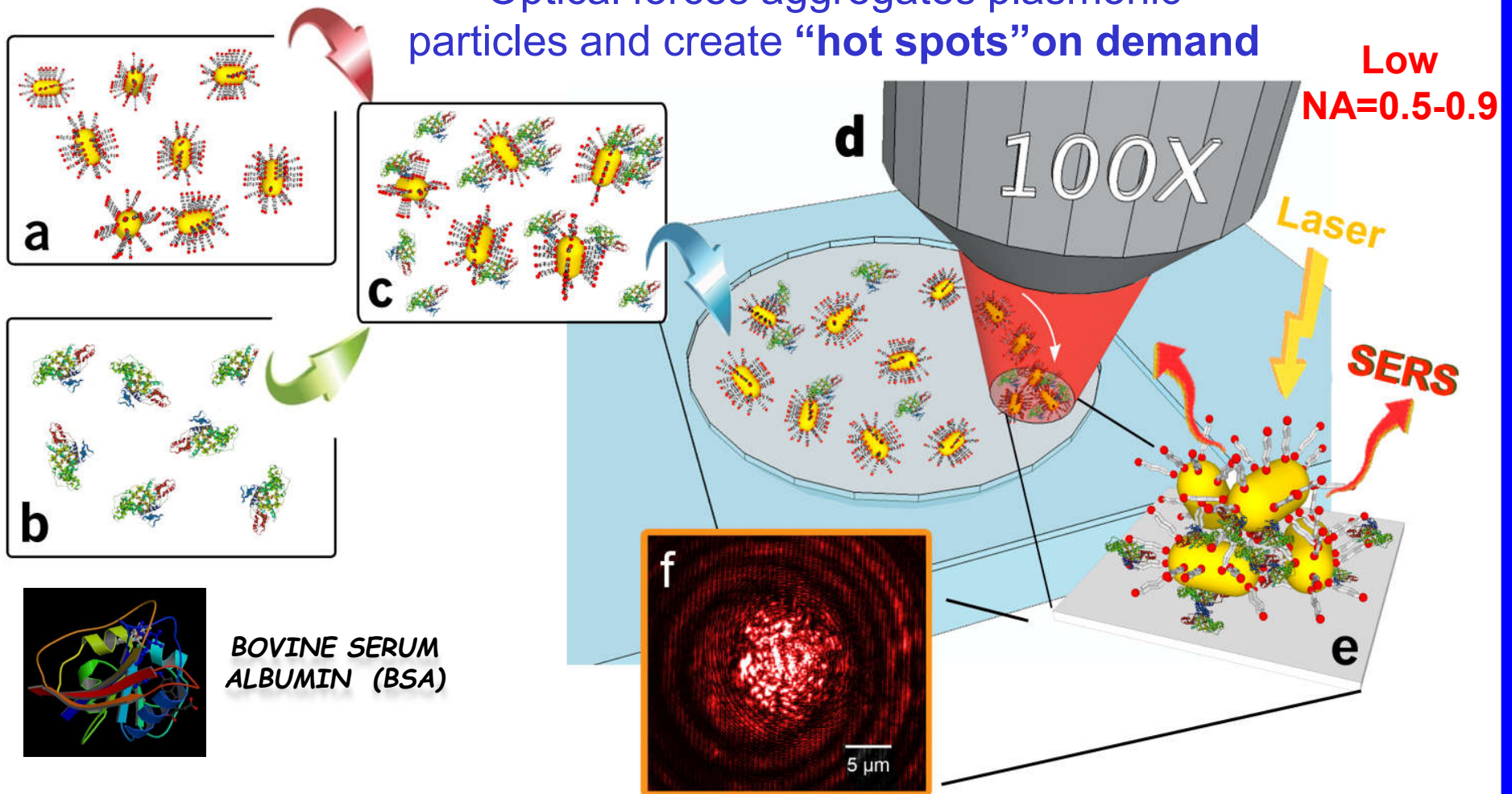
The huge electromagnetic field enhancement enables dramatic increase of the vibrational signal of molecules located in the *hot spots*.

A biosensor based on optical forces



Optical forces aggregates plasmonic particles and create “hot spots” on demand

Low
NA=0.5-0.9



Use the **SAME** light to push and excite SERS, Pushing wavelength $633\text{nm} < 687\text{nm}$ Nanorods SPR

Proteins (BSA) are detected directly in their **natural (PH)** liquid environment with high sensitivity

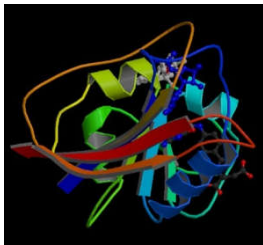
B. Fazio, et al., Scientific Reports 6 (2016): 26952



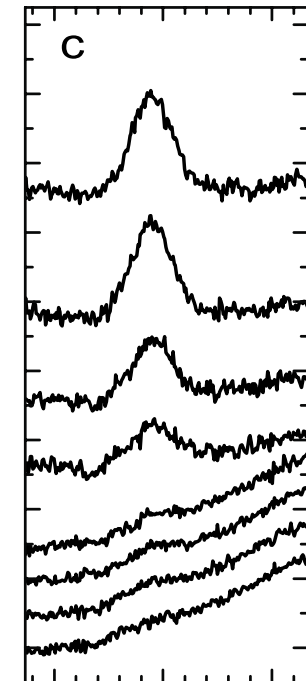
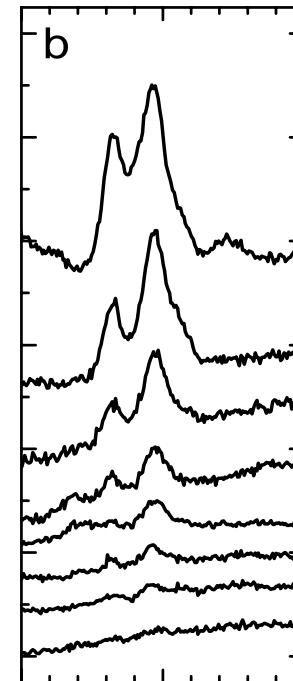
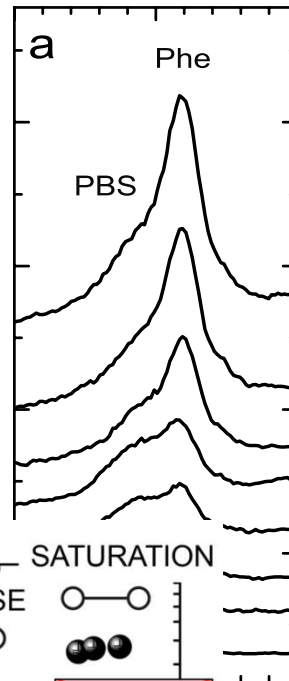
Time evolution of SERS spectra



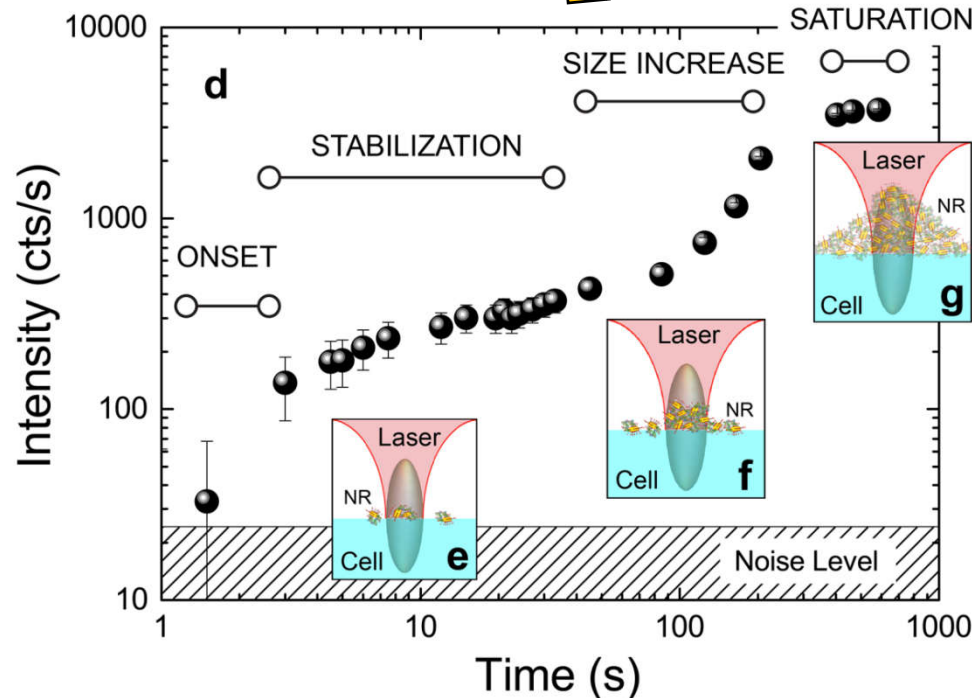
- a. Phe ring breathing
- b. Aromatic aminoacids and Amide I
- c. CH stretching region



Intensity (arb. units)



Time



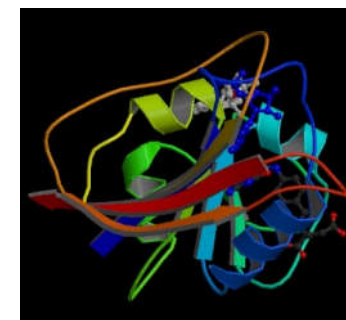
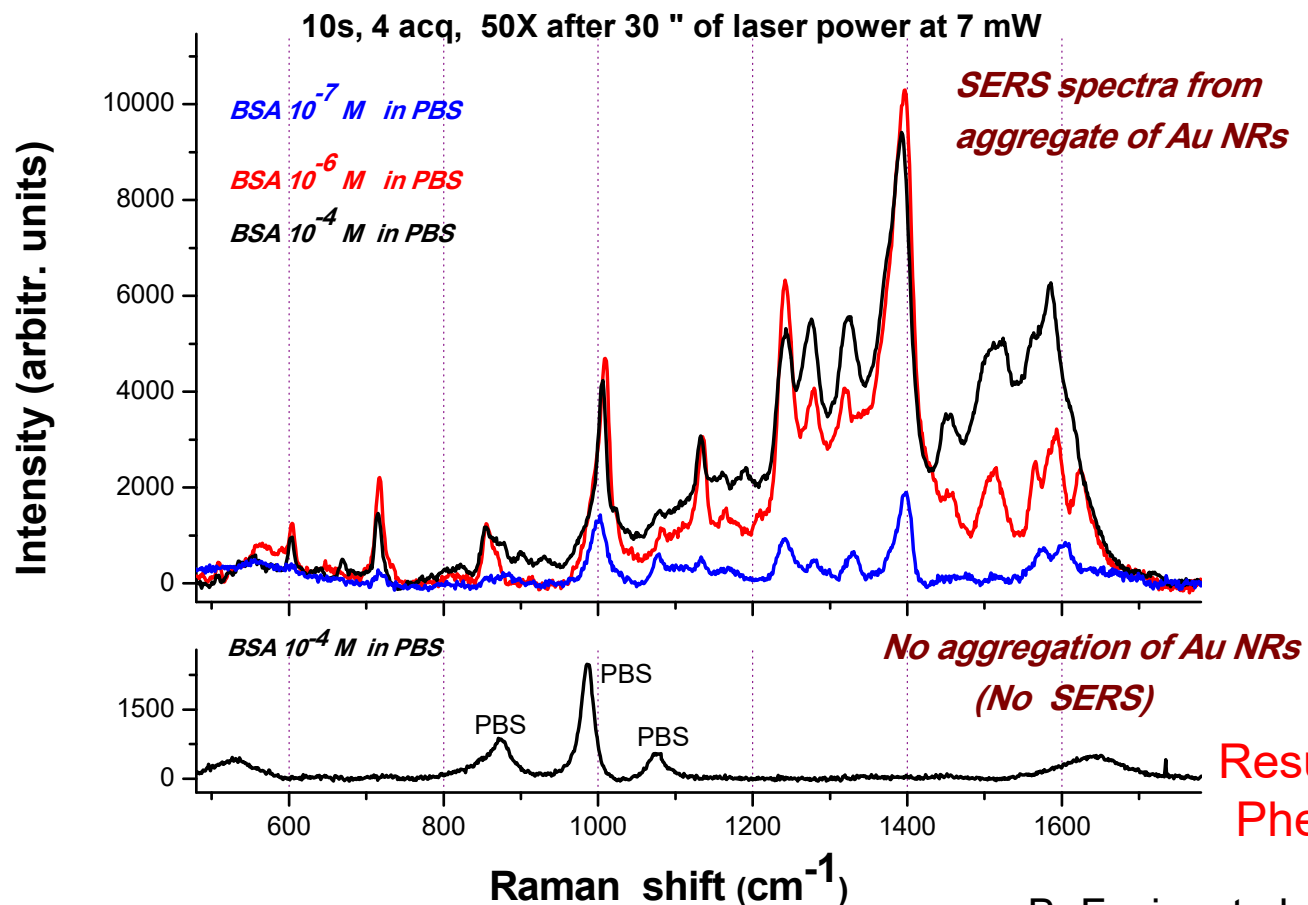
Raman shift (cm^{-1})

Time evolution of SERS signals can be used to study the dynamics of aggregation. This depends on wavelength and power.

SERS Detection at low molar concentration



Creation of HOT SPOT region in liquid environment for high sensitive spectroscopy

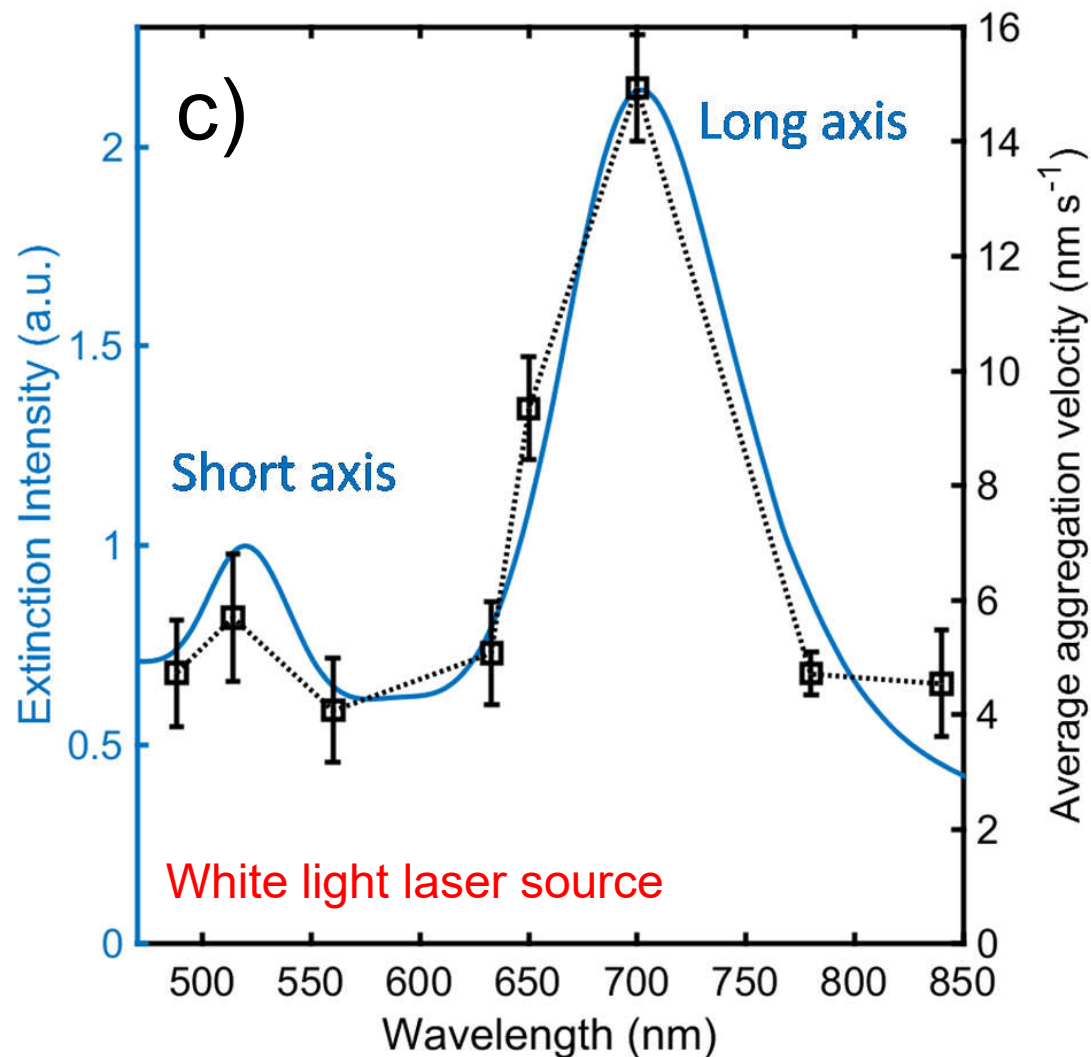
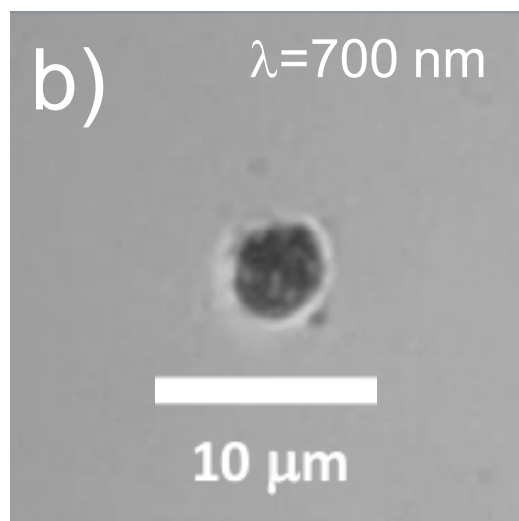
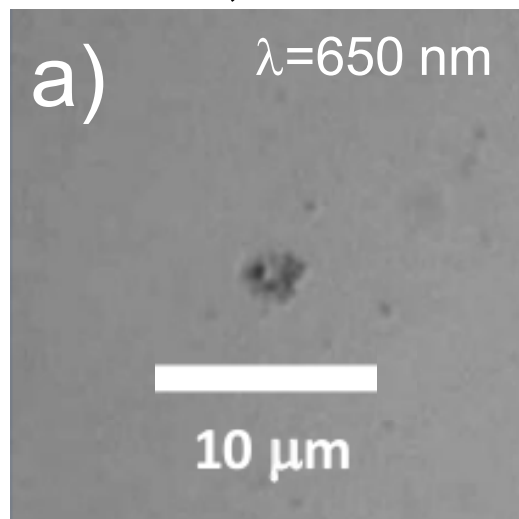


- Detection of **BSA** at concentrations as low as **10⁻⁷ M**
- Enhancement of BSA Raman scattering by **5 orders of magnitude**

Results also with Lysozyme, Phenylalanine, and Mnsod

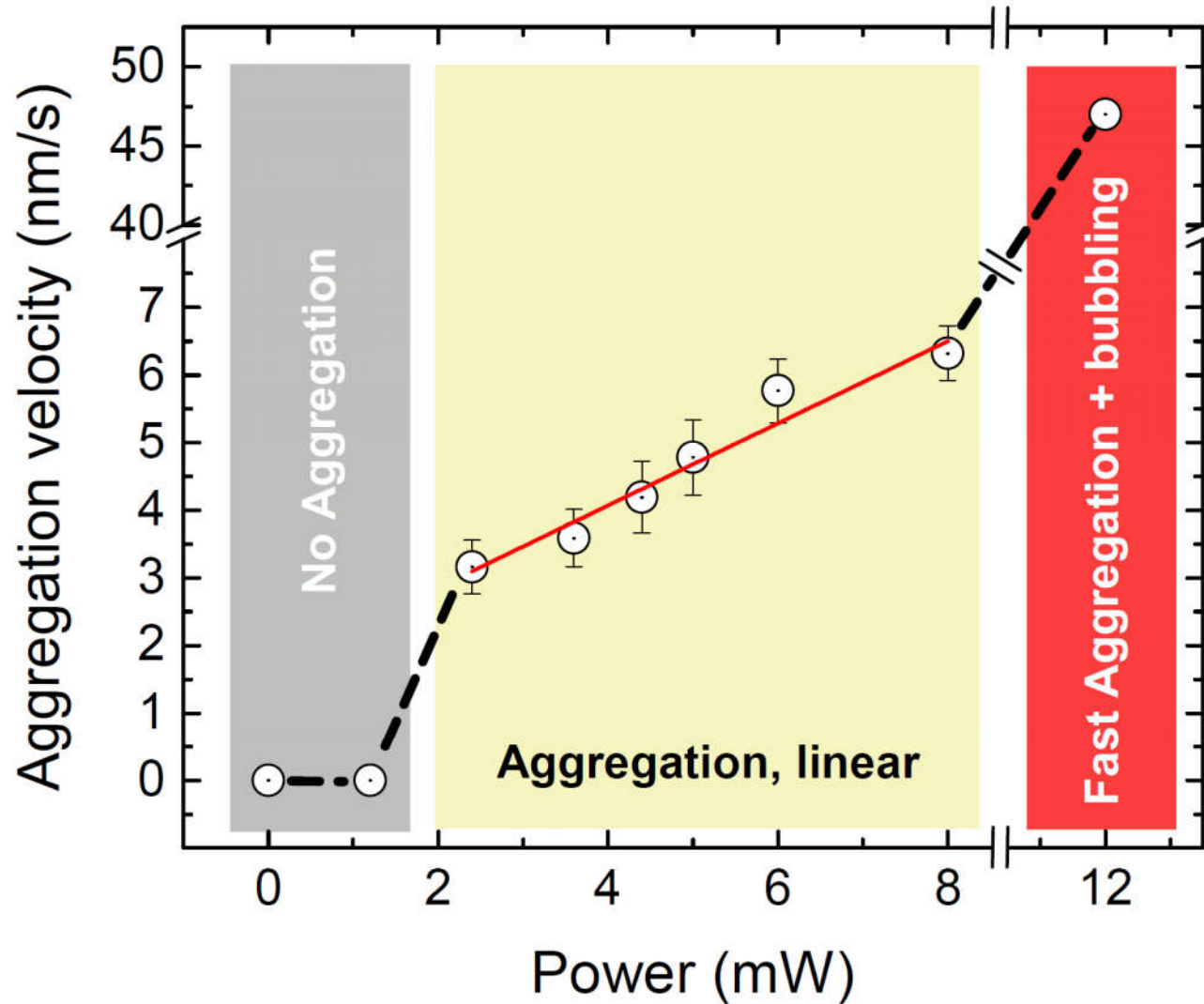
P=4 mW, after 500 s

S. Bernatova, et al., J. Phys. Chem C (2019)



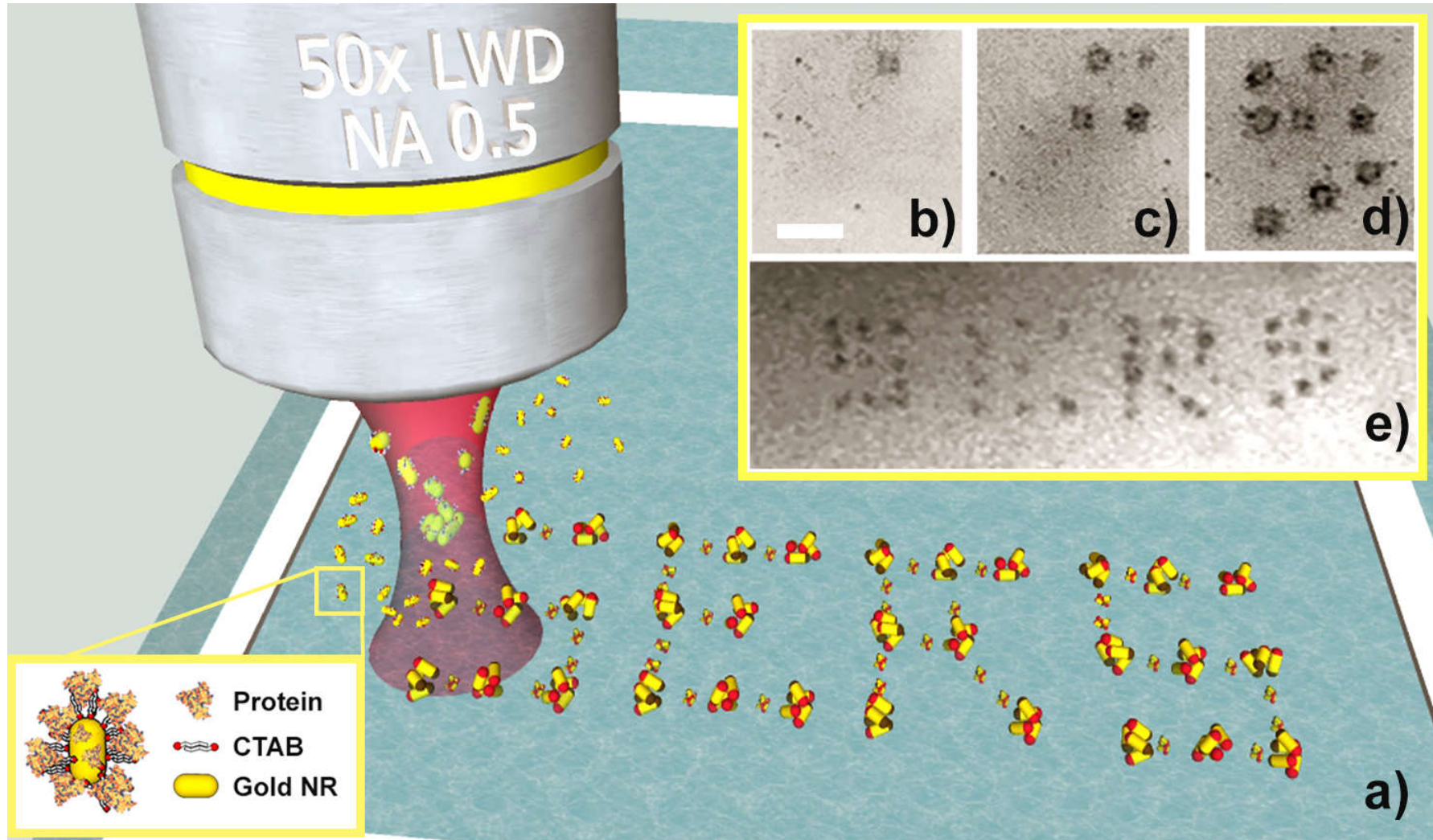
Average velocity of aggregation is estimated in the linear regime (before saturation) as the size of the aggregate divided by the aggregation time

Power dependence of aggregation velocity



S. Bernatova, et al., J. Phys. Chem C (2019)

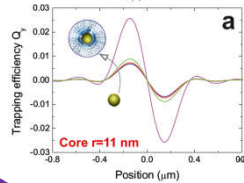
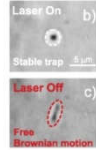
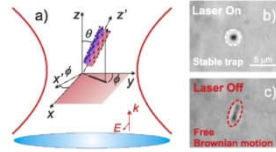
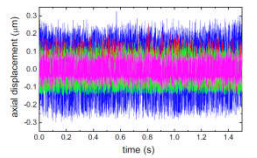
Controlled patterning for in situ SERS detection



A. Foti et al., *Materials* **11**, 440 (2018)

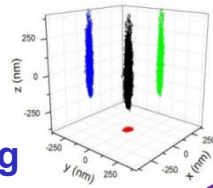
Optical forces

Calibration

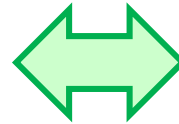


Theory

Scaling

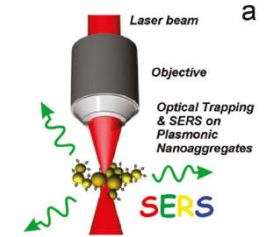
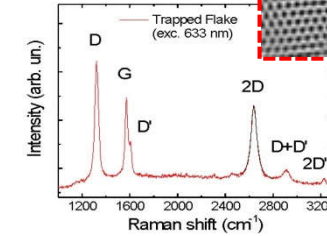


Raman Tweezers



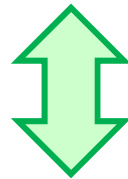
Spectroscopy

Graphene, Nanotubes metrology in LP



SERS Tweezers Biosensors

Novel Strategies



OT

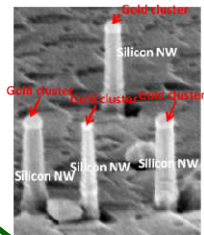
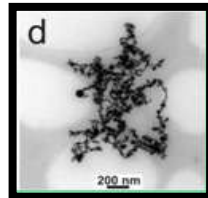
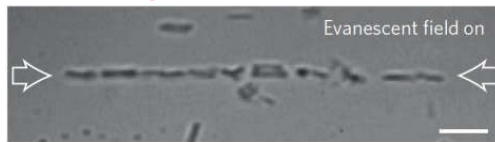


Optical Characterization

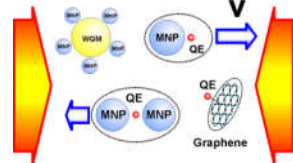
Shape

Aggregation

Binding

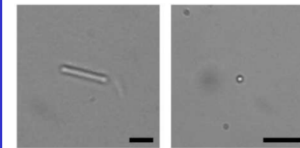


Hybridization

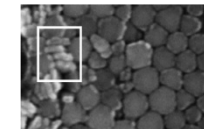


Materials

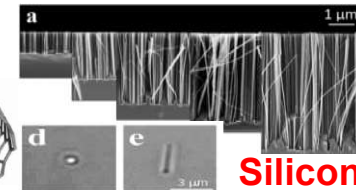
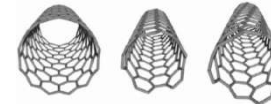
Nanofibers



Noble Metals

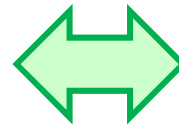


Carbon



Silicon

Quantum & Mesoscale

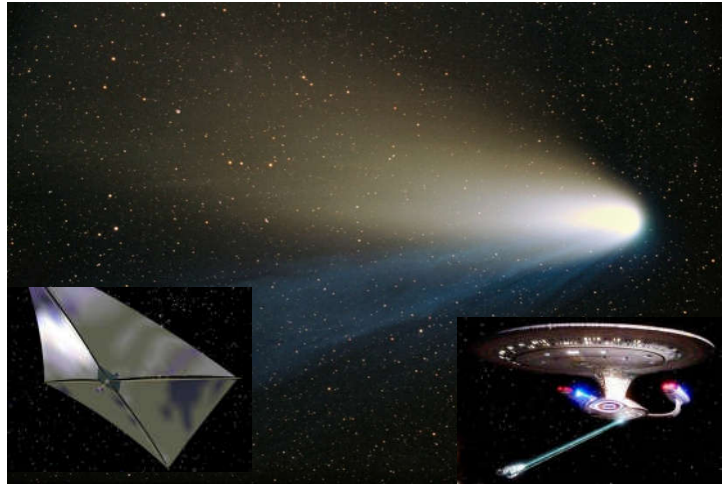




SPACE Tweezers



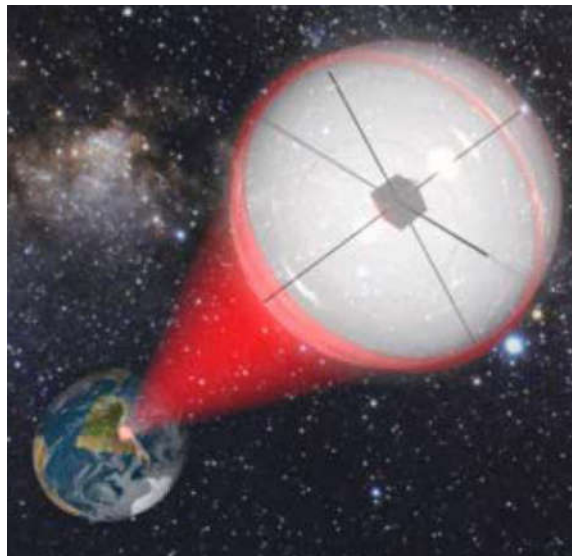
Optical forces were “born” in SPACE



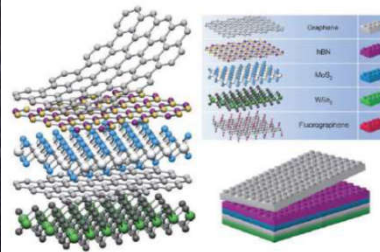
Bring them back “home”

Trap and investigate **nanoparticulate** matter in space or planetary atmospheres

Starshot project for a laser-driven lightsail



Nanomaterials and nanophotonic design



Atwater et al. Nat Mater. (2018)



Agenzia Spaziale Italiana

INAF

ISTITUTO NAZIONALE DI ASTROFISICA
NATIONAL INSTITUTE FOR ASTROPHYSICS

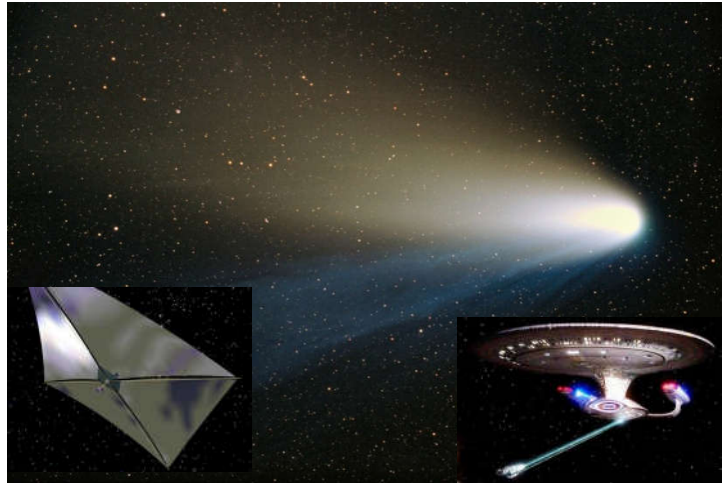
Edited by A. Magazzù

Thank You!



Sunset from Vulcano, Onofrio M. Maragò

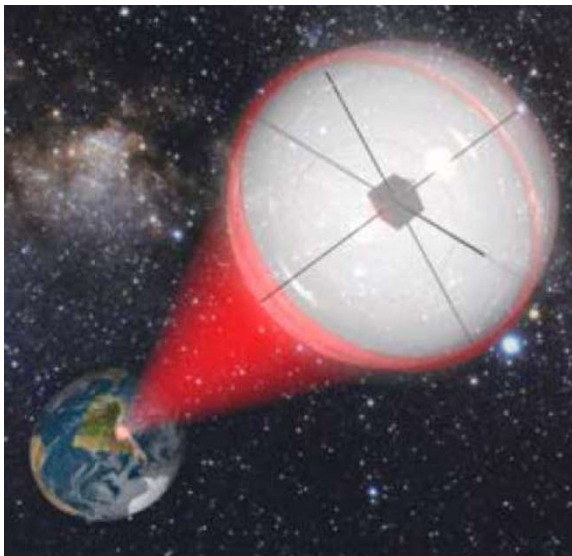
Optical forces were “born” in SPACE



Bring them back “home”

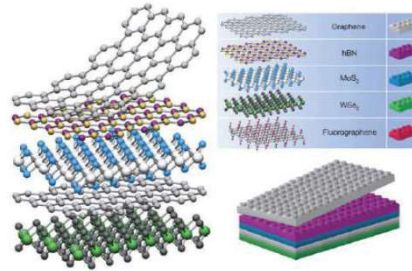
Trap and investigate **nanoparticulate** matter in space or planetary atmospheres

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Atwater et al. Nat Mater. (2018)

Nanomaterials and nanophotonic design



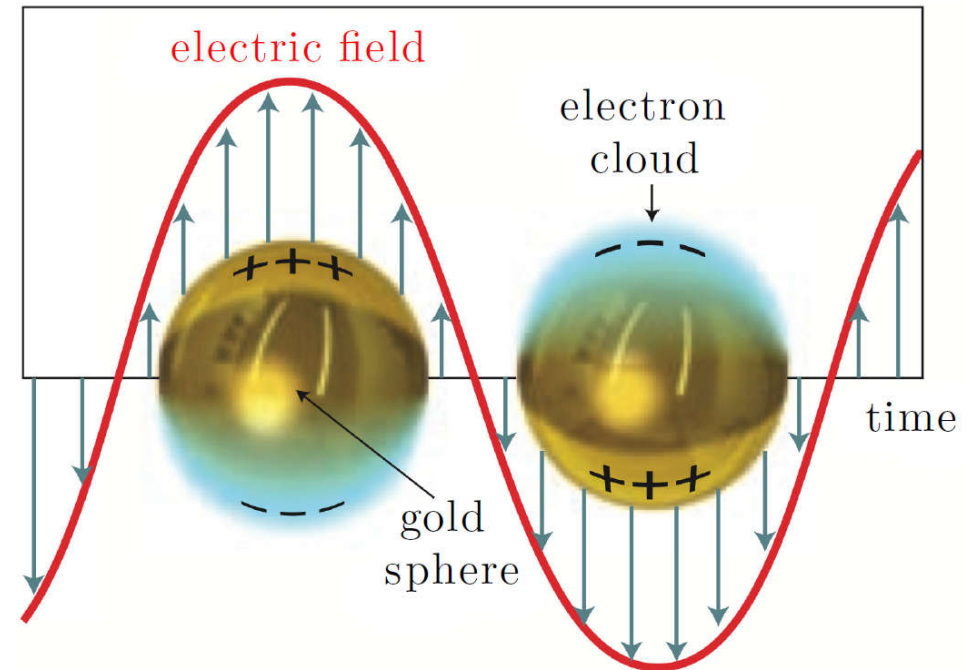
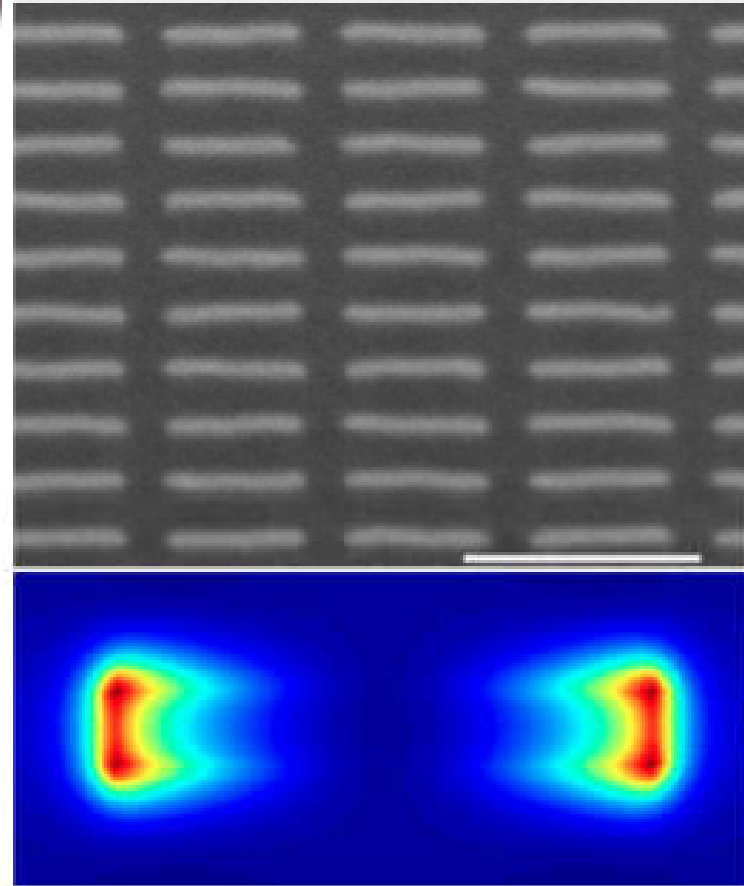
SPACE - TWEEZERS

INAF

Agenzia Spaziale Italiana
ISTITUTO NAZIONALE DI ASTROFISICA
NATIONAL INSTITUTE FOR ASTROPHYSICS



Localised surface plasmon



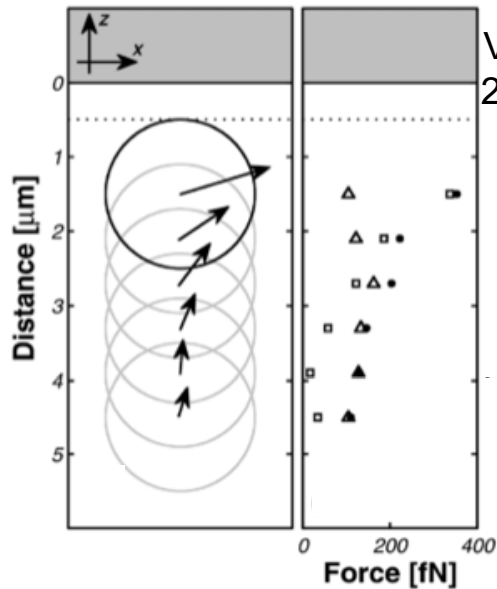
- ⇒ **Collective oscillation of the electrons inside the nanoparticle**
- ⇒ **Interaction with light**

Amendola, V., Pilot, R., Frasconi, M., Maragò, O. M., & Iatì, M. A. (2017). **Surface plasmon resonance in gold nanoparticles: a review.** *J. Phys.: Cond. Matt.*, 29, 203002.

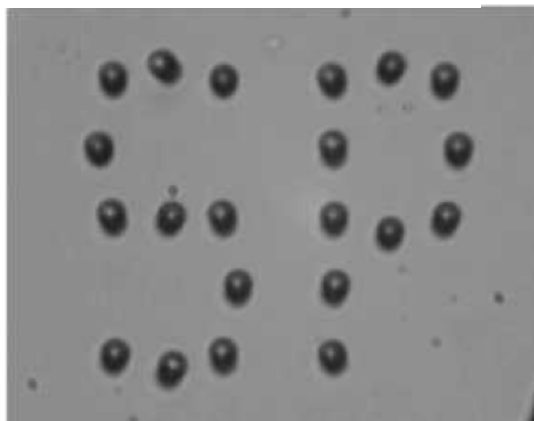
Juan et al., Nat. Photon. **5**, 349 (2011); Maragò et al., Nat. Nano. **8**, 807 (2013)

Surface Plasmon Polaritons

Plasmonic landscape (microscale)



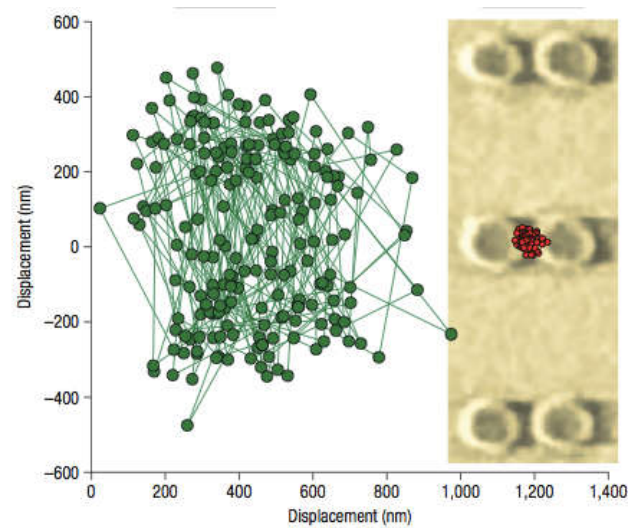
Volpe, PRL 96, 238101 (2006).



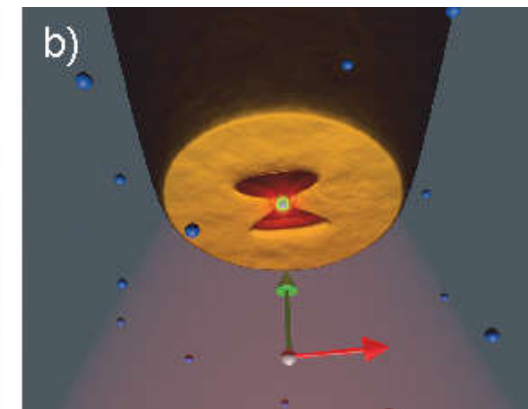
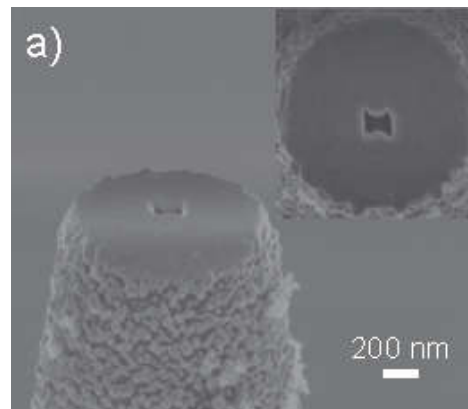
Righini, Nat. Phys. **3**, 477 (2007).

Localized Plasmon Polaritons

Nanoantennas (*hot spot* trapping, nanoscale)

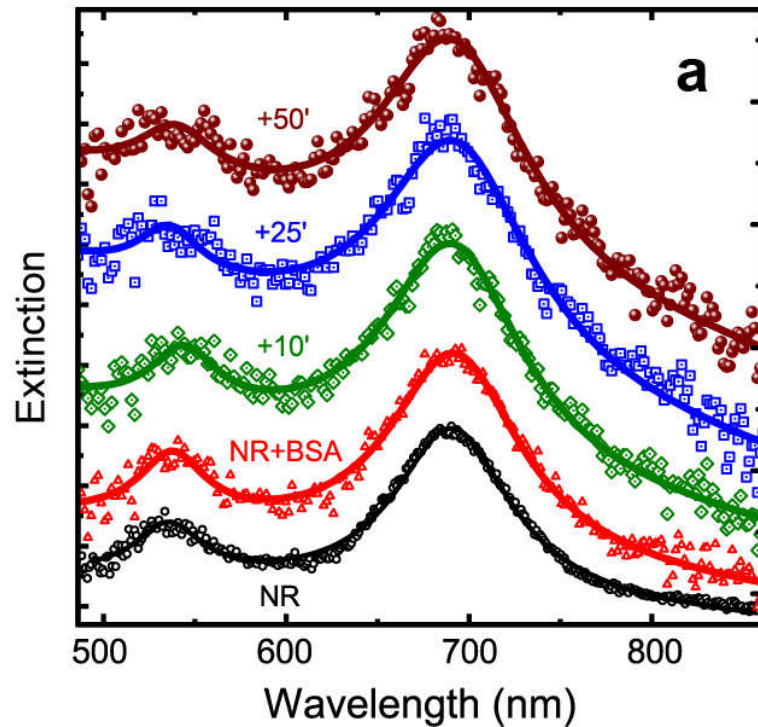


Grigorenko, Nat. Photon. **2**, 365 (2008).



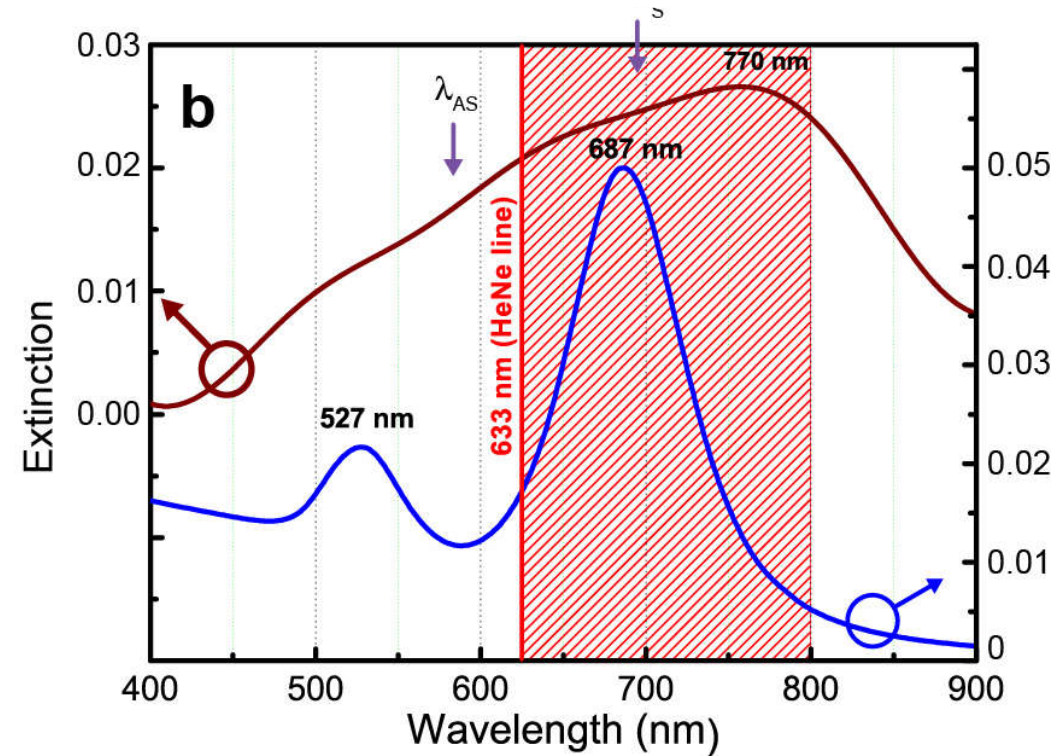
Berthelot, Nat. Nano. **9**, 295 (2014).

In solution UV spectra
do not change

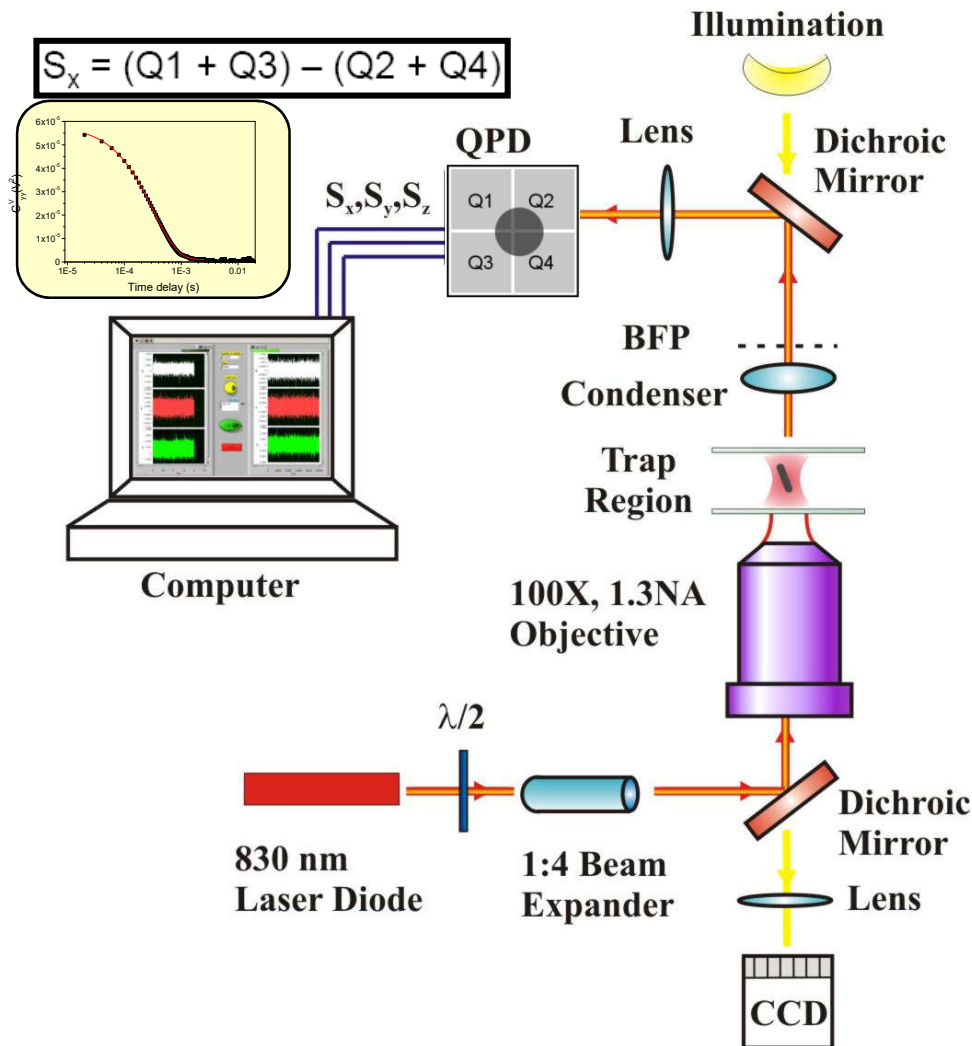


- ✓ **Protein-Nanorods complexes** in solution are composed by **individual NRs surrounded by protein layer.**

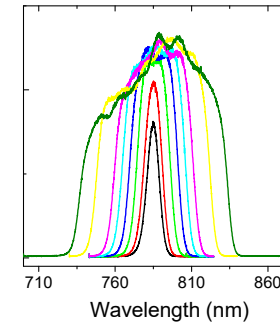
NRs are aggregated by optical forces
and **glued by plasmonic heating** at the
surface of the sample chamber



- ✓ **Broadening and red-shift** of the plasmon resonance, **nanorods are coupled.** This leads to enhanced fields, and then, SERS amplification of biomolecules spectra.



- Standard OT with QPD forward or back detection
- Multiwavelength: **830nm, 785nm**, 633nm, 417nm, **White Light Source**
- Radial Polarizer (arcoptics)
- Piezostage (1nm resolution)
- LC waveplate
- Galvomirrors



Back focal plane interferometry combined with a QPD is sensitive to Brownian fluctuations

Brownian motion is a key ingredient in Force Sensing with optical tweezers.

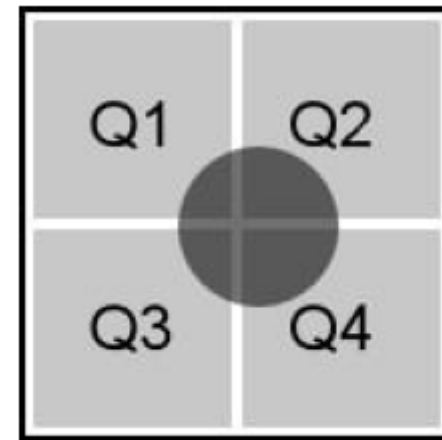
NIR light ensures very low water absorption



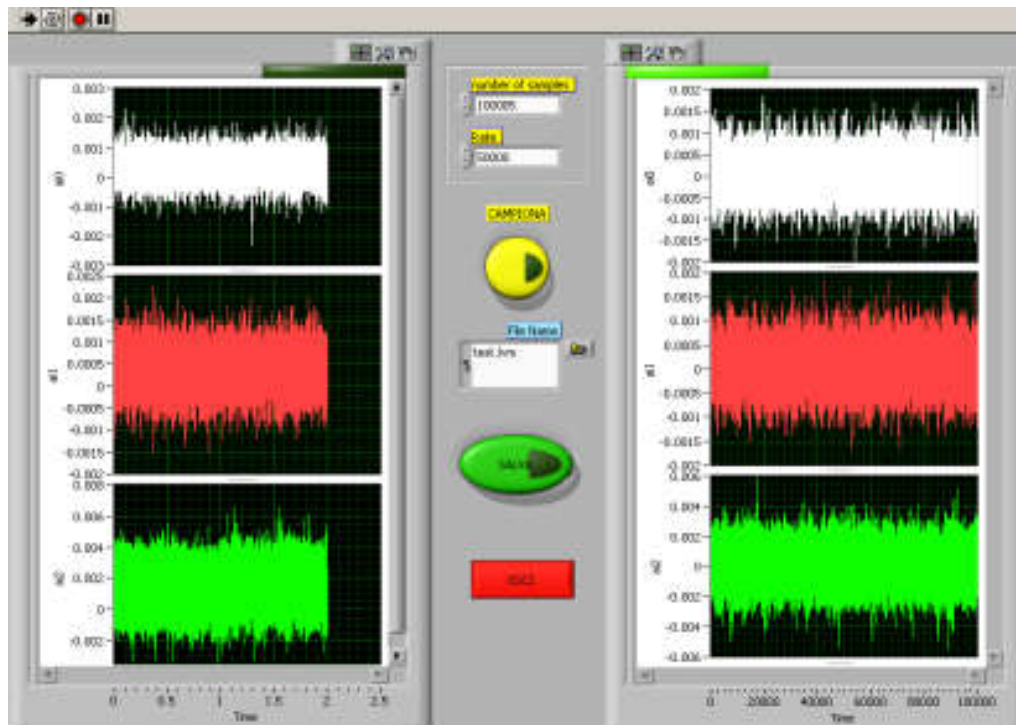
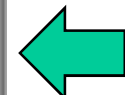
IPCF How do we measure forces?



We measure the interference pattern between scattered and unscattered light in the back focal plane on a QPD. Time evolution is proportional to the particle positional fluctuations.



$$S_x = (Q1 + Q3) - (Q2 + Q4)$$



Brownian motion is the key ingredient to calibrate Optical Tweezers.

- Equation of motion of a damped harmonic oscillator subject to a randomly fluctuating force:

$$m \frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \kappa x = \xi(t)$$

Stokes
Trap

- The term $\xi(t)$ describes random (uncorrelated) fluctuations in force with zero mean, i.e.

$$\langle \xi(t) \rangle = 0 \quad \langle \xi(t + \tau) \xi(t) \rangle = \frac{2k_B T}{\gamma} \delta(\tau)$$

- Equation of motion in the overdamped regime:

$$\gamma \partial_t x(t) = -\kappa x(t) + \xi(t)$$

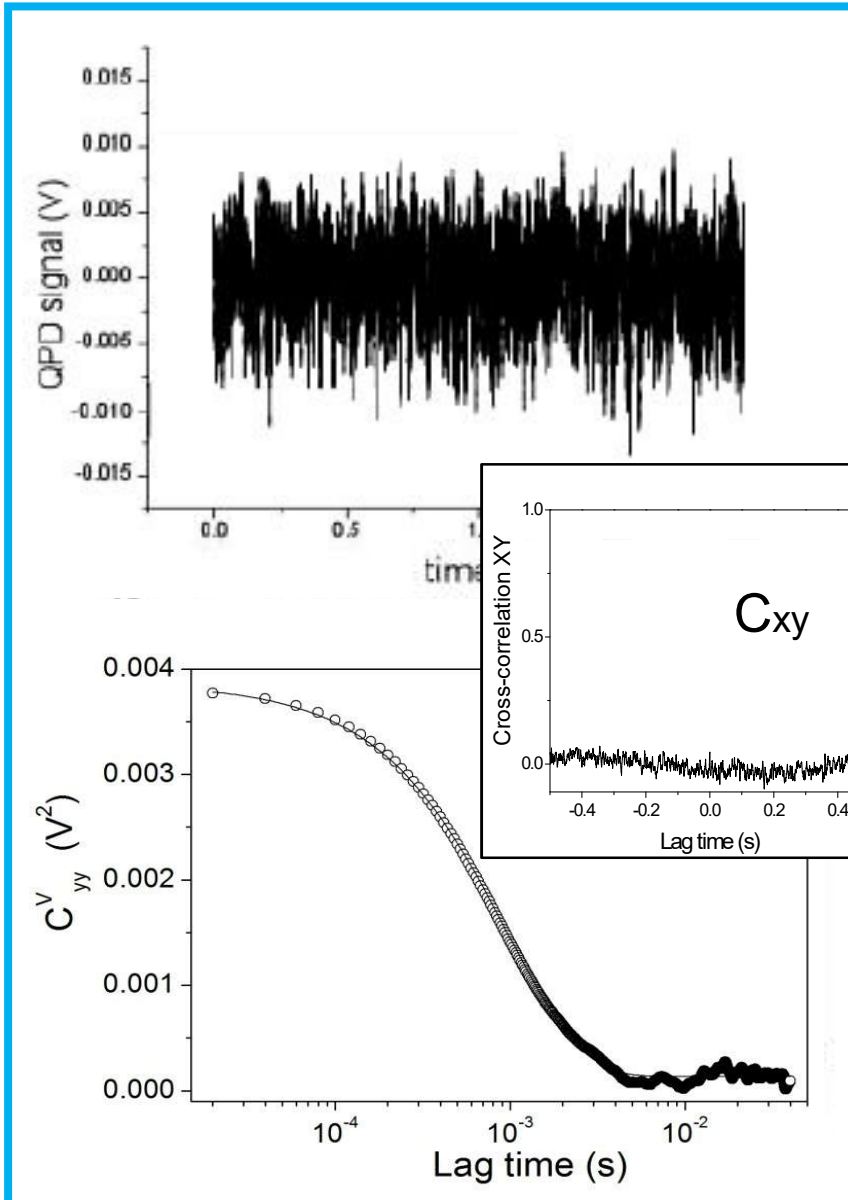
- Calculate the **autocorrelation** of position fluctuations:

$$C_{xx}(\tau) = \langle x(t)x(t + \tau) \rangle$$

- The solution to which is straightforward:

$$C_{xx}(\tau) = C_{xx}(\tau = 0) \exp(-\omega\tau)$$

$$\omega = \frac{\kappa}{\gamma}$$



From QPD tracking signals we get Autocorrelation Functions and eventually the Force Constants

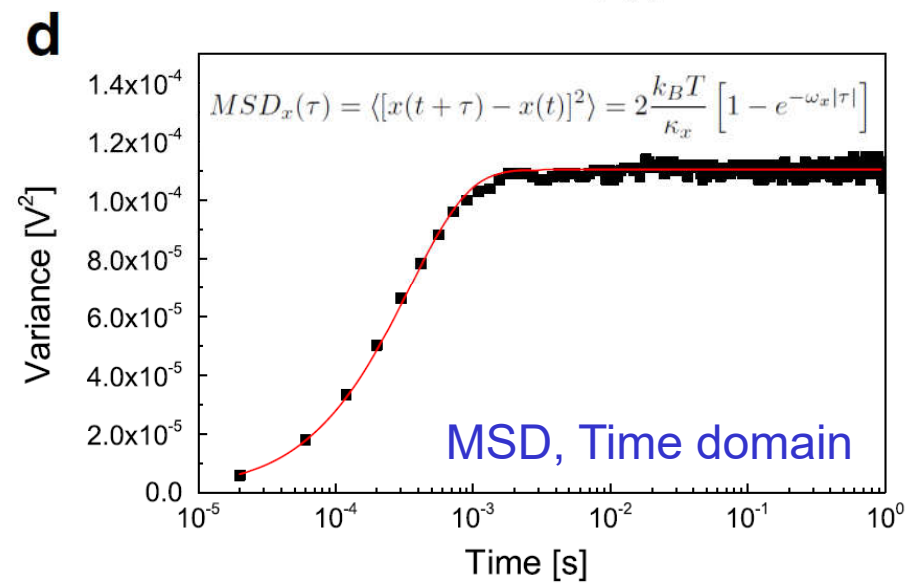
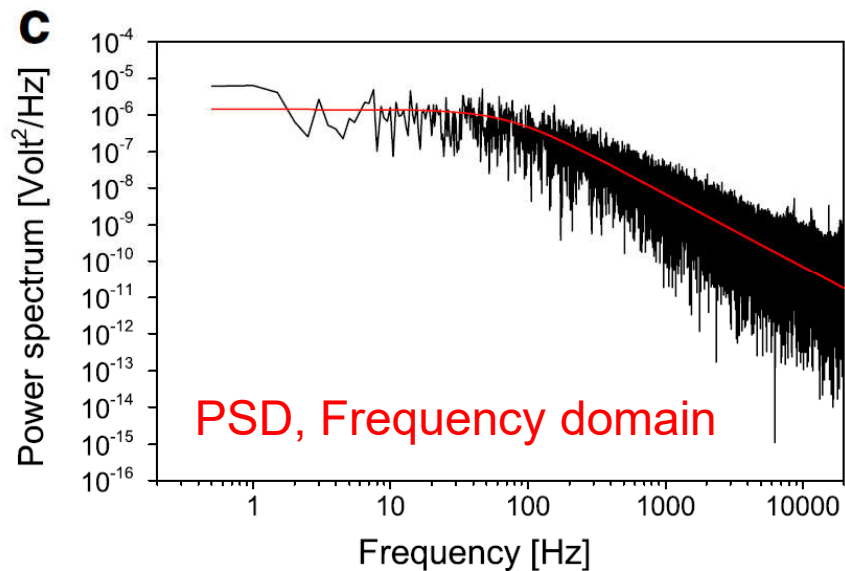
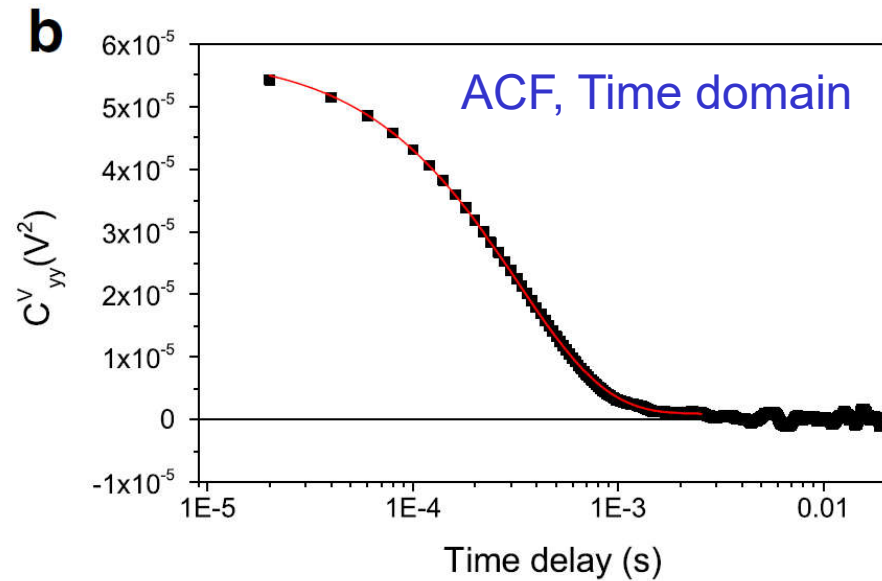
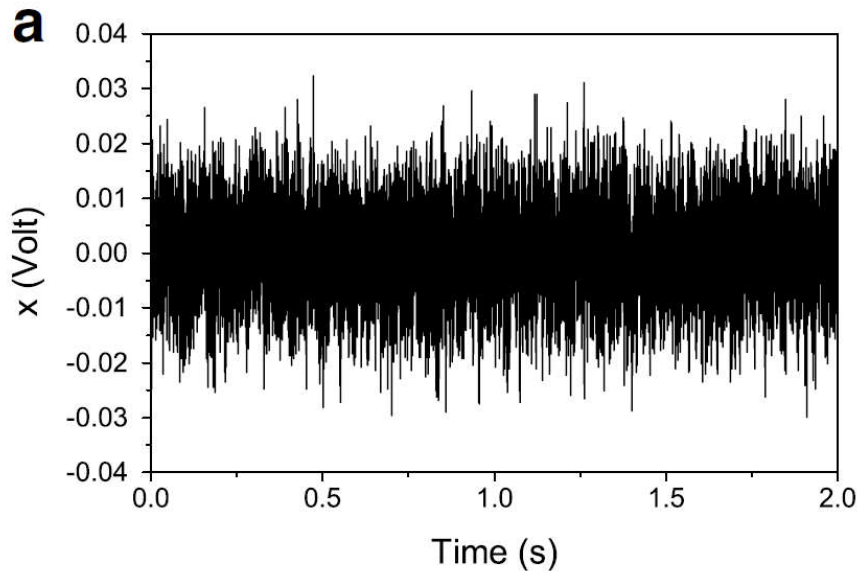
$$C_{xx}(\tau) = C_{xx}(0) \exp\left(-\frac{\kappa_x}{\gamma} \tau\right)$$

$$C_{xx}^V(\tau) = \langle V_x(t)V_x(t + \tau) \rangle = \beta_x^2 C_{xx}(\tau)$$

$$C_{xx}^V(0) = \beta_x^2 C_{xx}(0) = \beta_x^2 k_B T / \kappa_x$$

Calibration factor

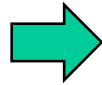
$$\beta_x = \sqrt{\frac{C_{xx}^V(0) \kappa_x}{k_B T}}$$



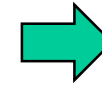
Optical trap potential analysis



Dynamics



Probability

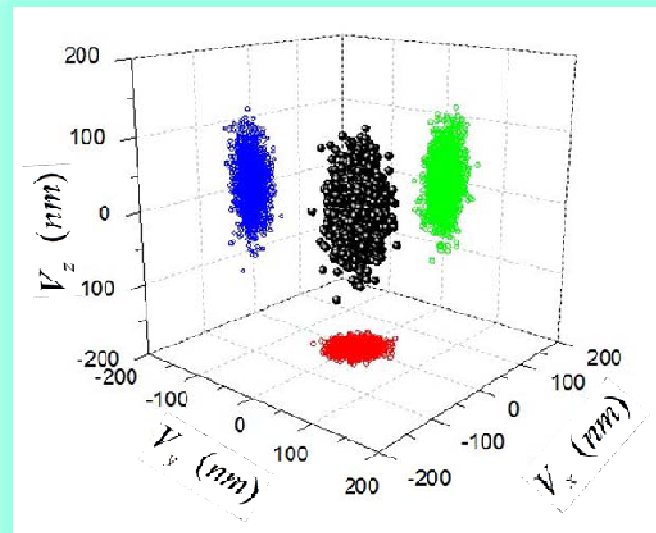
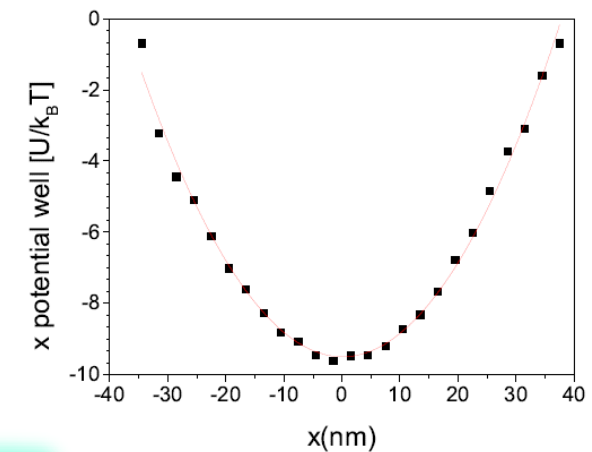
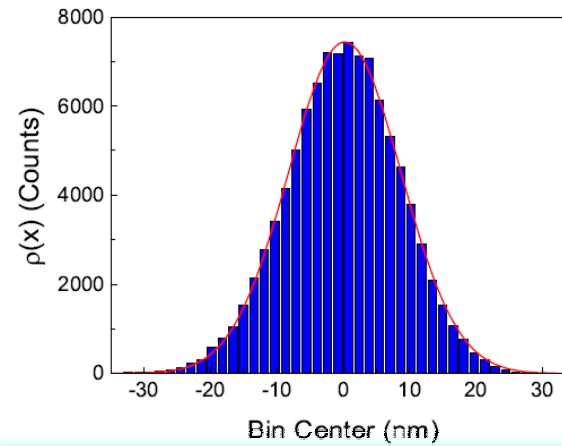
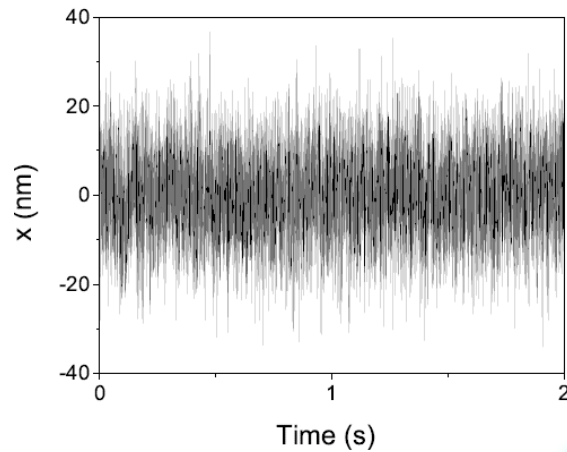


Potential

$$\frac{d}{dt}x(t) = -\frac{1}{\gamma} \frac{d}{dx}U(x) + \sqrt{2D}W_x(t)$$

$$\rho(x) = \rho_0 \exp \left[-\frac{U(x)}{k_B T} \right]$$

$$U(x) = -k_B T \log [\rho(x)] + U_0$$



From each signal we can reconstruct the effective trapping potential in 3D

Brownian Motion is more complex

$$\begin{aligned}\partial_t X_i(t) &= -\omega_i X_i(t) + \xi_i(t), \quad i = x, y, z \\ \partial_t \Theta_j(t) &= -\Omega_j \Theta_j(t) + \xi_j(t), \quad j = x, y\end{aligned}$$

From correlation functions we can extrapolate the force and torque constants on the SWNT bundle

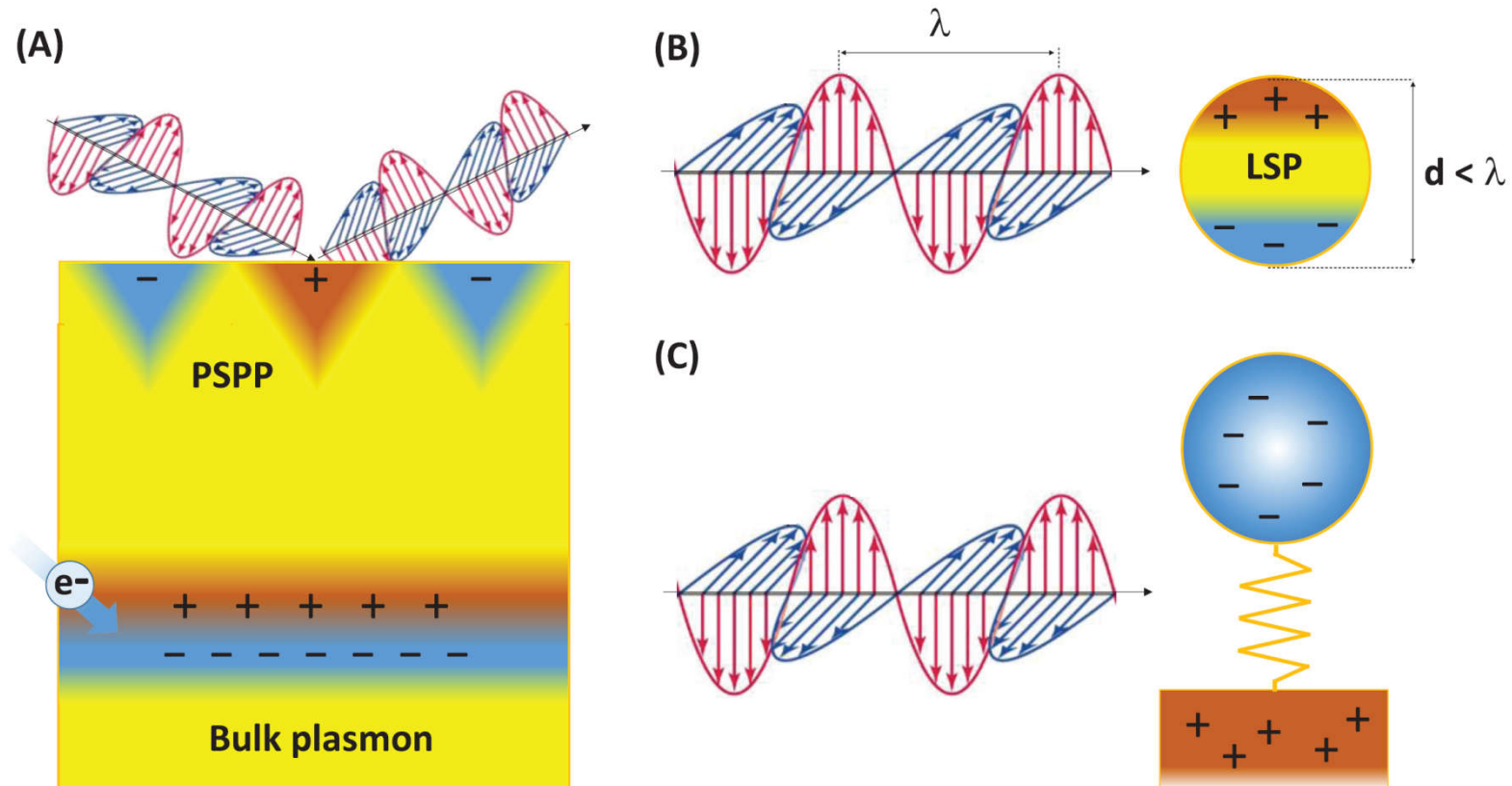
$$\begin{aligned}C_{X_i X_i}(\tau) &= \langle X_i(t) X_i(t + \tau) \rangle \\ C_{\Theta_j \Theta_j}(\tau) &= \langle \Theta_j(t) \Theta_j(t + \tau) \rangle\end{aligned}$$

$$\begin{aligned}\omega_x &= \Gamma_{\perp} k_x, \quad \omega_y = \Gamma_{\perp} k_y, \quad \omega_z = \Gamma_{\parallel} k_z \\ \Omega_x &= \Gamma_{\Theta} k_{\Theta_x}, \quad \Omega_y = \Gamma_{\Theta} k_{\Theta_y}.\end{aligned}$$

Relaxation Frequencies for Translational and Angular Motion

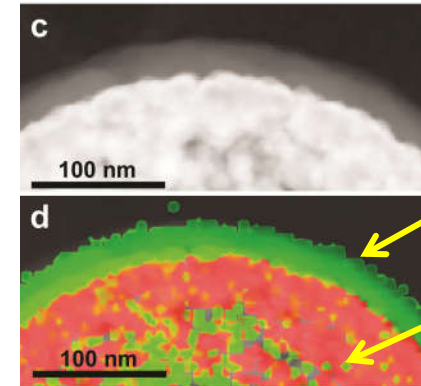
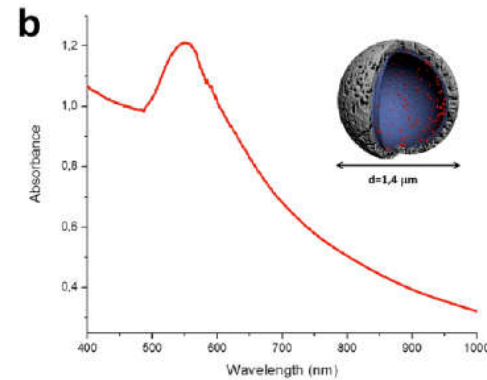
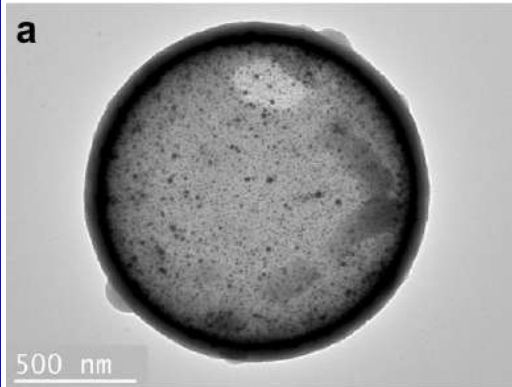
Hydrodynamics of a rod-like nanostructure is embedded in the relaxation frequencies

Surface Plasmon Polaritons and Localized Surface Plasmons

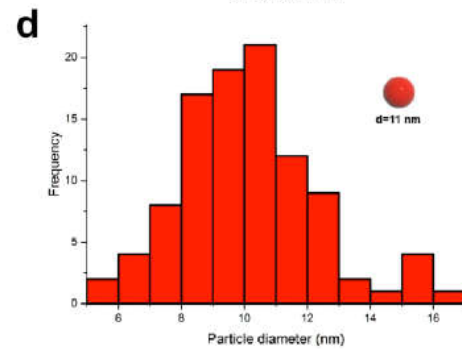
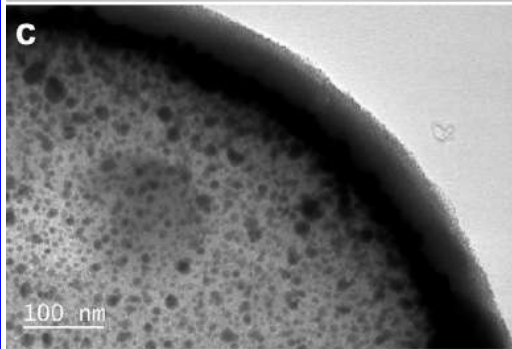


Amendola, V., Pilot, R., Frasconi, M., Maragò, O. M., & Iatì, M. A. (2017). Surface plasmon resonance in gold nanoparticles: a review. *J. Phys.: Cond. Matt.*, 29(20), 203002.

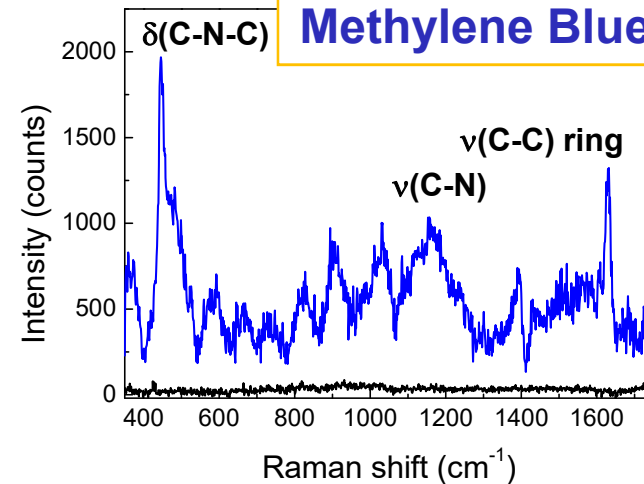
Mesocapsule: external porous **Silica** shell **30nm**, internal **AuNPs** with diameters **11nm**. Capsule diameter **1.4 microns**



Silica
Au



SERS Tweezers
Methylene Blue 10⁻⁵ M



Cargo mesocapsules can be manipulated and activated to release specific molecules in-situ

Spadaro et al. J Phys Chem C (2017)

SERS enhancement due to AuNPs in the inner walls of Mesocapsule, that can be reached by MB molecules thanks to the porosity of the shell.

Optical Trapping in Dipole Approximation

- Particle size parameter is small, $x = k_m a \ll 1$
- Interaction of electric field of laser with induced dipole in dielectric $\mathbf{p}(\mathbf{r}, t) = \alpha_p \mathbf{E}(\mathbf{r}, t)$
- The (harmonic) trapping potential is defined by the incident light intensity

$$\langle \mathbf{F} \rangle_{\text{DA}} = \underbrace{\frac{1}{2} \frac{n_m}{c \epsilon_m} \Re \{ \alpha_p \} \nabla I(\mathbf{r})}_{\text{Gradient force}} + \underbrace{\frac{n_m}{c} \sigma_p \mathbf{E}(\mathbf{r})}_{\text{Scattering force}}$$

$$U_{\text{dip}} = -\underline{p} \cdot \underline{E}$$

$$\underline{F}_{\text{grad}} \propto \underline{\nabla} I(\underline{r}) = -\kappa_i x_i$$

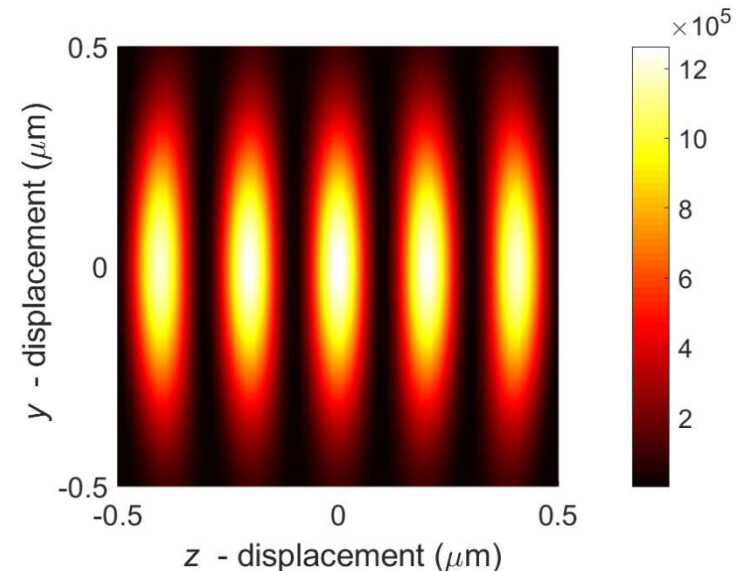
J. R. Arias-González and M. Nieto-Vesperinas, *JOSA A* (2003)

Counter-propagating Gaussian beam

$$\kappa_{\rho}^{\text{c.p.}} = 8 \frac{\Re \{ \alpha_p \} I_0}{c n_m w_0^2}, \quad z_0 = \frac{k_m w_0^2}{2}, \quad w_0 = 0.5 \lambda_0 / NA$$

$$\kappa_z^{\text{c.p.}} = 4 \frac{\Re \{ \alpha_p \}}{c n_m} (2 - 2k_m z_0 + k_m^2 z_0^2) \frac{I_0}{z_0^2}, \quad I_0 = 2P / \pi w_0^2$$

O. Brzobohatý et. al. *Opt. Exp.* (2015)



Resonant gain metal/dielectric nanoshell

Under a pumping threshold, nano-shell shows a stable dipolar field.

$$\bar{\mathbf{p}} = \frac{\alpha_{\text{NUM}}(\epsilon_1, \epsilon_2, \epsilon_3, \rho)}{\alpha_{\text{DEN}}(\epsilon_1, \epsilon_2, \epsilon_3, \rho)} \bar{\mathbf{E}}$$



$$\rho = \frac{a_1}{a_2}$$

a_1 int. radius

a_2 ext. radius

$$\alpha = a_2^3 \frac{(\epsilon_2 - \epsilon_3)(\epsilon_1 + 2\epsilon_2) + \rho^3(\epsilon_1 - \epsilon_2)(\epsilon_3 + 2\epsilon_2)}{(\epsilon_2 + 2\epsilon_3)(\epsilon_1 + 2\epsilon_2) + 2\rho^3(\epsilon_2 - \epsilon_3)(\epsilon_1 - \epsilon_2)}$$

ϵ_3 water
(solvent)
dielectric
permittivity

Steady State Gain dielectric permittivity

$$\epsilon_1 = \epsilon_b - \frac{G\Delta}{2(\omega - \omega_{21}) + i\Delta} \quad \Delta = \frac{2}{\tau_2}$$

ϵ_b dielectric host permittivity

$$G = \Im[\epsilon_1(\omega_{21})] = -\frac{n\mu^2\tau_2}{3\hbar\epsilon_0} \tilde{N}$$

Single metallic nanoparticle permittivity
(Drude model)

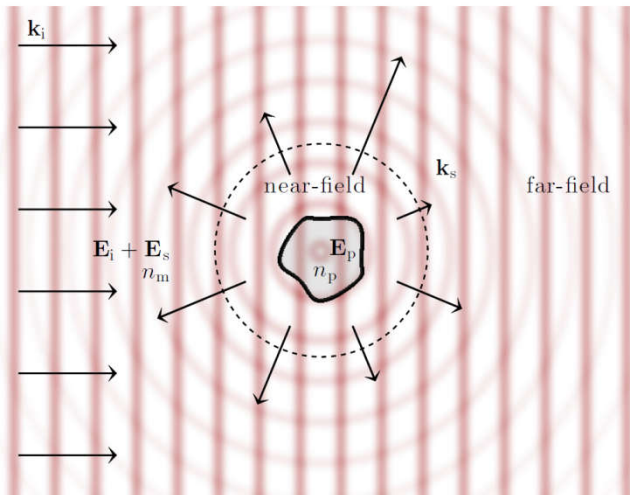
$$\epsilon_2 = \epsilon_\infty - \frac{\omega_{pl}^2}{\omega(\omega + 2i\gamma)}$$

In our calculation,

$\rho = 0,77$ and $a_2 = 20$ nm



The Scattering Problem – Multipole Expansion



Helmholtz equations

$$(\nabla^2 + n^2 k_v^2) \mathbf{E} = 0, \quad (\nabla^2 + n^2 k_v^2) \mathbf{B} = 0$$

Boundary conditions

$$\mathbf{E}_2 = \mathbf{E}_i + \mathbf{E}_s \quad \mathbf{E}_1 = \mathbf{E}_p$$

$$\hat{\mathbf{n}} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0, \quad \hat{\mathbf{n}} \times (\mathbf{B}_2 - \mathbf{B}_1) = 0$$

Expansion of the incident field

$$\mathbf{E}_i(r, \hat{\mathbf{r}}) = E_i \sum_{l=0}^{\infty} \sum_{m=-l}^l W_{i,lm}^{(1)} \mathbf{J}_{lm}^{(1)}(r, \hat{\mathbf{r}}) + W_{i,lm}^{(2)} \mathbf{J}_{lm}^{(2)}(r, \hat{\mathbf{r}})$$

Expansion coefficients

$$\mathbf{J}_{lm}^{(1)} = j_l(kr) \mathbf{X}_{lm}(\hat{\mathbf{r}})$$

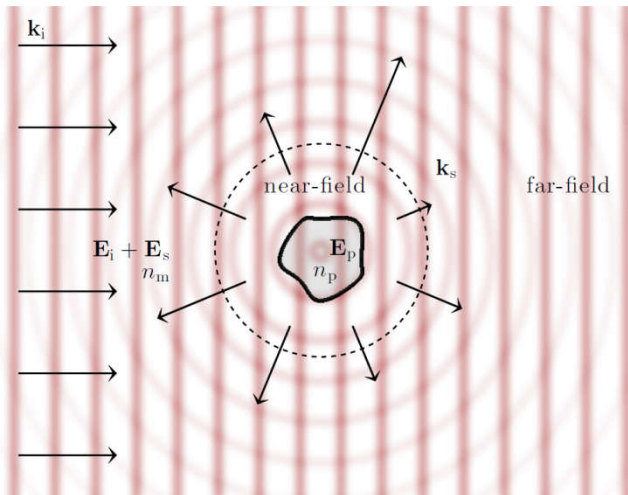
Magnetic multipole

$$\mathbf{J}_{lm}^{(2)} = \frac{1}{k} \nabla \times \mathbf{J}_{lm}^{(1)}$$

Electric multipole

Vector spherical harmonics

$$\mathbf{X}_{lm} = [l(l+1)]^{-1/2} \mathbf{L} Y_{lm}$$



Expansion of the scattered wave

$$\mathbf{E}_s(r, \hat{\mathbf{r}}) = E_i \sum_{l=0}^{\infty} \sum_{m=-l}^l A_{s,lm}^{(1)} \mathbf{H}_{lm}^{(1)}(r, \hat{\mathbf{r}}) + A_{s,lm}^{(2)} \mathbf{H}_{lm}^{(2)}(r, \hat{\mathbf{r}})$$

Expansion of the internal field

$$\mathbf{E}_p(r, \hat{\mathbf{r}}) = E_i \sum_{l=0}^{\infty} \sum_{m=-l}^l W_{p,lm}^{(1)} \mathbf{J}_{lm}^{(1)}(r, \hat{\mathbf{r}}) + W_{p,lm}^{(2)} \mathbf{J}_{lm}^{(2)}(r, \hat{\mathbf{r}})$$

Boundary conditions

$$\hat{\mathbf{n}} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0$$

$$\mathbf{E}_2 = \mathbf{E}_i + \mathbf{E}_s$$

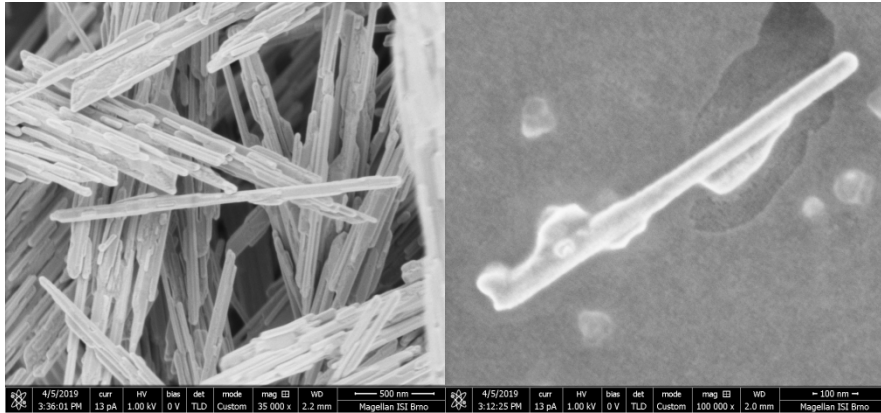
$$\mathbf{E}_1 = \mathbf{E}_p$$

By imposing the Boundary conditions at the particle surface it is possible to find the relation between the **A** and **W** coefficients

$$A_{s,l'm'}^{(p')} = \sum_{p=1,2} \sum_{l=0}^{\infty} \sum_{m=-l}^l T_{lml'm'}^{(pp')} W_{i,lm}^{(p)}$$

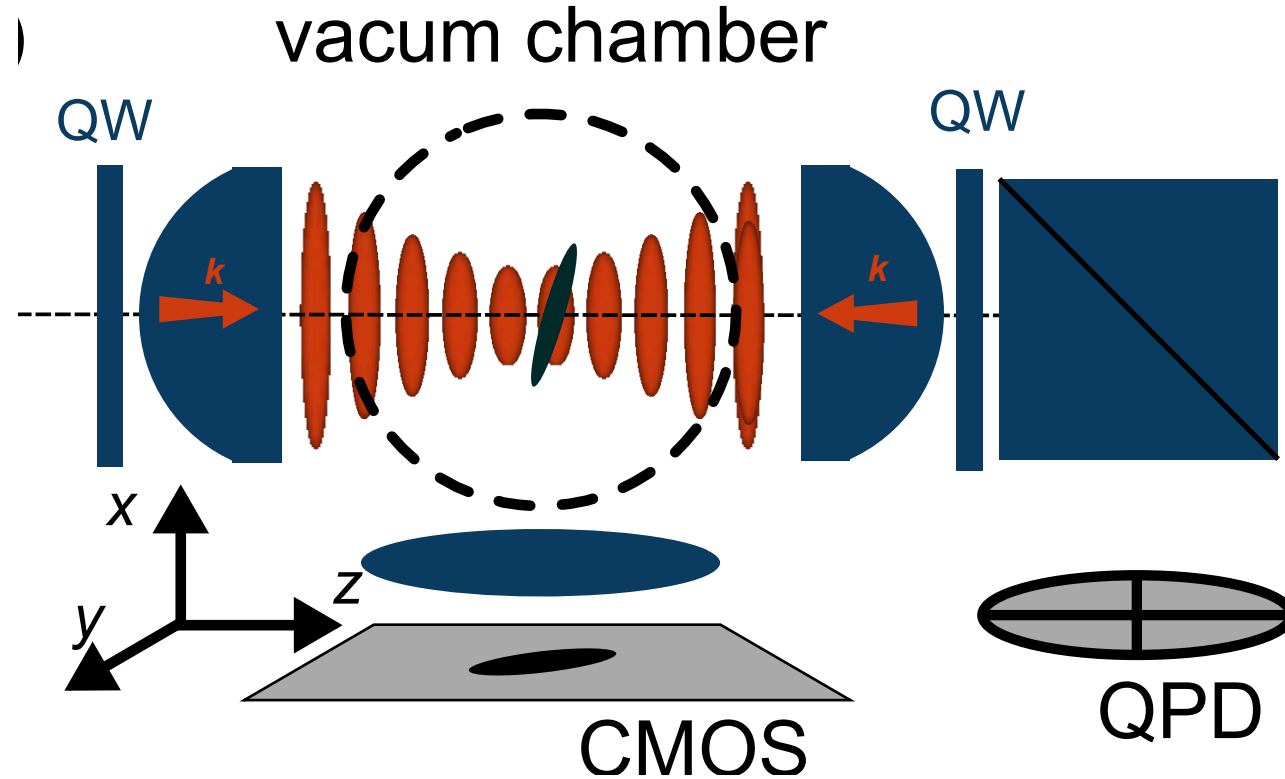
$$\mathbf{E}_s = \mathbf{T} \mathbf{E}_i$$

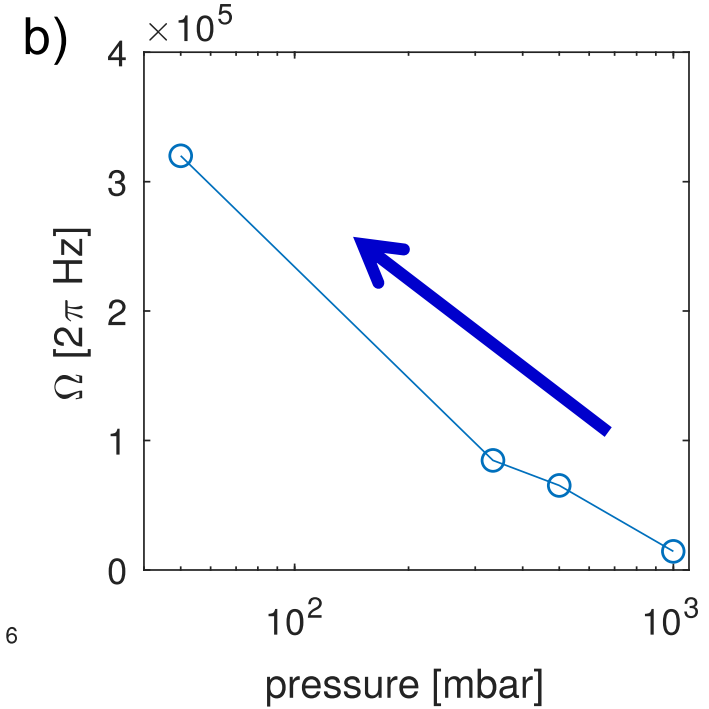
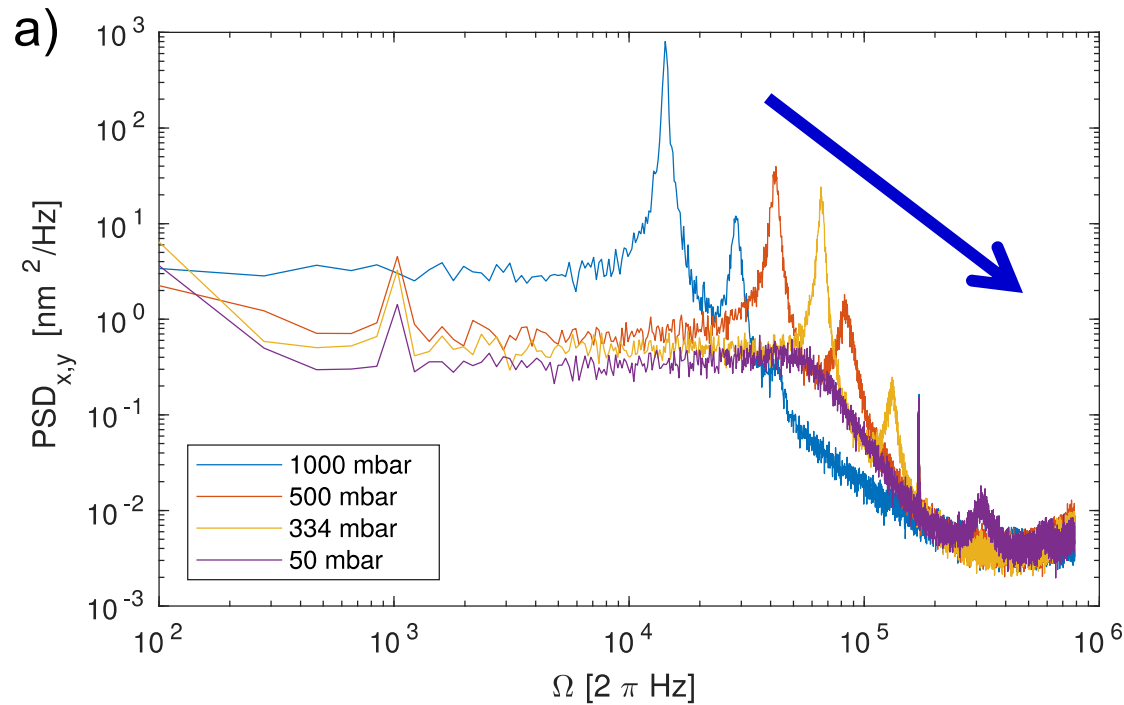
T- Matrix



Commercial samples
 $L=1$ micron, $d=70$ nm

$NA=0.5$, $w_0 = 2.8 \mu\text{m}$ beam waist





Translations (background)

Rotations

$$\text{PSD}(\Omega) = \frac{k_B T}{\pi m} \left[\frac{\Gamma_{\text{transl}}}{(\Omega_{\text{transl}}^2 - \Omega^2)^2 + \Gamma_{\text{transl}}^2 \Omega^2} + \sum_i \frac{\Gamma_{\text{rot},i}}{(\Omega_{\text{rot},i}^2 - \Omega^2)^2 + \Gamma_{\text{rot},i}^2 \Omega^2} \right]$$

Damping decreases with pressure, hence rotational frequency increases

Length controls both transferred torque and damping